

summary: overall population has mean  $\mu$ , sd or (unknown).

SRS of size  $n$  has sample mean  $\bar{x}$ , standard deviation  $s$ .

$$\text{standard error } SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

the standardized sample mean

$\frac{\bar{x} - \mu}{s/\sqrt{n}}$  has a t-dist w/  $n-1$  degrees of freedom.

A level C confidence interval is  $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ , where  $t^*$  is the value for which  $(-t^*, t^*)$  gives area  $C$  under the  $t_{n-1}$  curve margin of error.

Significance tests:  $H_0: \mu = \mu_0$  calculate  $\frac{\bar{x} - \mu_0}{s/\sqrt{n}}$  and compare with  $t^*$ .

## §7.2 Comparing two means

Take two SRS from two populations - do they have the same mean?

Population	variable	mean	sd	sample size	sample mean	sample sd.
1	$x_1$	$\mu_1$	$\sigma_1$	$n_1$	$\bar{x}_1$	$s_1$
2	$x_2$	$\mu_2$	$\sigma_2$	$n_2$	$\bar{x}_2$	$s_2$

don't know!

Consider: estimate  $\mu_1 - \mu_2$ , by  $\bar{x}_1 - \bar{x}_2$ .

Variance of  $\bar{x}_1 - \bar{x}_2$  is  $\text{var}(\bar{x}_1) + \text{var}(\bar{x}_2) = \frac{\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{n_2^2}$

Assume populations both normal  $N(\mu_1, \sigma_1)$ ,  $N(\mu_2, \sigma_2)$  and  $\sigma_1, \sigma_2$  known.

then the two sample z statistic is  $z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{n_2^2}}}$   $\sim N(0, 1)$

Suppose we don't know  $\sigma_1, \sigma_2$ :

two sample t statistic is  $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1^2} + \frac{s_2^2}{n_2^2}}}$ . approx  $t_k$   
(for significance test  $\beta$ ). when  $k = \min\{n_1-1, n_2-1\}$

two sample t confidence interval  $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1^2} + \frac{s_2^2}{n_2^2}}$

$t^*$  comes from  $t_k$   
 $k = \min\{n_1-1, n_2-1\}$