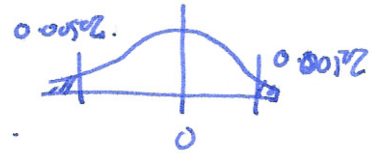
p-value is $1-\text{pnorm}(4.02) = \text{pnorm}(-4.02) << 0.01$

significant, reject H_0, ~~accep~~ significant evidence that $\mu \neq 15$. (at $\alpha = 0.01$ significance level).

Q: what is the power of this test? Against a specific alternative 15.5?

$H_0: \mu = 15$

$H_a: \mu \neq 15$

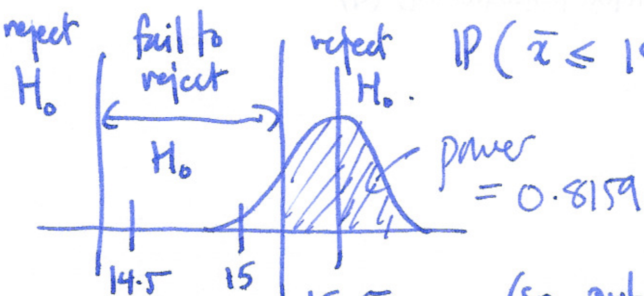


Q: when does the test reject H_0? when $|z| \geq 2.576$.

test statistic is $z = \dfrac{\bar{x} - 15}{0.25/\sqrt{3}}$. i.e. $\bar{x} \geq 15.37$, $\bar{x} \leq 14.63$

assume $\mu = 15.5$, then $\mathbb{P}(\bar{x} \geq 15.37) = \mathbb{P}\left(\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} \geq \dfrac{15.37 - 15.5}{0.25/\sqrt{3}}\right)$

$= \mathbb{P}(z \geq -0.90) = 0.8159.$



$\mathbb{P}(\bar{x} \leq 14.63) = \mathbb{P}\left(\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} \leq \dfrac{14.63 - 15.5}{0.25/\sqrt{3}}\right)$

$= \mathbb{P}(z \leq -6.03) = 0$

(so prob type II error is $1 - 0.8159 = 0.1841$).

80% power considered good.

how to increase power: • ~~decrease~~ increase sample size &

§7.1 Inference for means (no longer assume we know σ!).

typical situation: population has mean μ s.d. σ. (we don't know!).

take an SRS of size n sample mean $\bar{x}$ sample s.d. s.

estimate for s.d. of $\bar{x}$ is called standard error $SE_{\bar{x}} = \dfrac{s}{\sqrt{n}}$

recall if we take an SRS of size n from an $N(\mu,\sigma)$ population, (σ known!)
then the $\bar{x}$ has dist $N(\mu, \sigma/\sqrt{n})$. statistic is $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$.

key fact if we take an SRS of size n from an $N(\mu,\sigma)$ population (σ not known!)
then the (one)-sample t statistic is

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

not normally distributed! (more spread out).

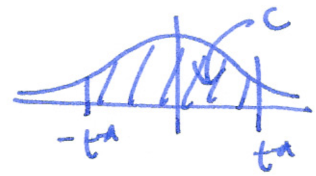
has a <u>t distribution with $(n-1)$-degrees of freedom.</u>

(for large n, this is close to $N(0,1)$).

confidence intervals take an SRS of size n from a population with mean μ, s.d. σ

a <u>level C confidence interval</u> is $\bar{x} \pm \underbrace{t^* \frac{s}{\sqrt{n}}}_{\text{margin of error}}$.

where t^* is the value for the $t(n-1)$ density curve w/ area C between $-t^*$ and t^*



(this is exactly ~~corr~~ right when pop is $N(\mu,\sigma)$ and approx. right for any dist. when n large).

Example watching TV: Nielsen claims average is 18.5 hrs/week. (age 18-24)

take sample of size 8: 3.0, 16.5, 10.5, 40.5, ~~8~~ 5.5, 33.5, 0.0, 6.5.

sample mean $\bar{x} = 14.5$

sample s.d $s = 14.854$

standard error $SE_{\bar{x}} = s/\sqrt{n} = 14.854/\sqrt{8} = 5.252$

find 95% confidence interval: degree of freedom $n-1=7$

t^*	1.895	2.365	2.517
C	0.9	0.95	0.96

$t^* = 2.365$.

so ~~$\bar{x}$~~ confidence interval is $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

$$= 14.5 \pm 2.365 \frac{14.854}{\sqrt{8}} = (2.08, 26.92).$$