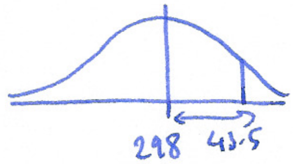
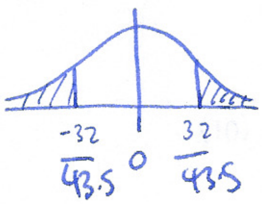
~~alternative hypothesis H_a = the means are different.~~

(2) A:



sample mean $\bar{x}$ has dist. $N\left(298, \frac{435}{\sqrt{100}}\right)$.

how likely is it that $\bar{x}$ differs from 298 by $\geq |298-262| = 32$?



$z = \frac{32}{43.5}$ probability (p-value) $= 2\,\text{pnorm}\left(\frac{-32}{43.5}\right) = 0.4620$

46%.

fairly likely! not much evidence that means are different.

Summary (significance testing).

(1) state null hypothesis: H_0 (e.g. no difference in means)

alternative hypothesis: H_a (e.g. means are different)

(2) calculate test statistic: $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$.

(3) find p-value: prob $X \sim N(0,1)$ has $|X| \geq z$.

(4) draw conclusion: choose a significance level α (e.g. $\alpha = 5\%$)

and reject H_0 if p-value $\leq \alpha$.

i.e. only 5% chance this difference arises from chance

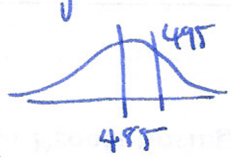
Example test to see if a sample comes from a population with given mean μ_0

Average SAT scores are 485, with sd = 100.

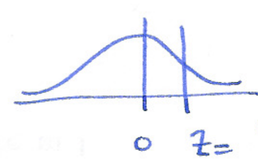
Average SAT score of 500 incoming freshmen (srs) at CBT are 495.

Q: is this evidence they have higher SAT score at $\alpha = 0.1$ significance?

sample mean $\bar{x} \sim N\left(485, \frac{100}{\sqrt{500}}\right)$.

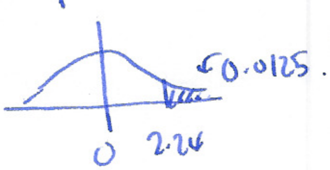


$\leadsto$



$z = \frac{495-485}{100/\sqrt{500}} = 2.24$

test statistic: $\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$.



(a 1-sided test!)

p-value is 0.0125, reject H_a at 10% level.