The variance of a random variable. (measure of spread)

value of X	x_1	x_2	$\cdots$
prob.	p_1	p_2	$\cdots$

$$\mu_X = \sum_i x_i p_i$$

$$\sigma_X^2 = \sum (x_i - \mu_X)^2 p_i$$

(average value of square of difference from mean)

standard deviation is $\sigma_X = \sqrt{\sigma_X^2}$.

example toss coin

	0	1
	$1/2$	$1/2$

$$\mu_X = 0 \cdot \tfrac{1}{2} + 1 \tfrac{1}{2} = \tfrac{1}{2}.$$

$$\sigma_X^2 = (0 - 1/2)^2 \tfrac{1}{2} + (1 - 1/2)^2 \cdot \tfrac{1}{2}.$$

$$= \tfrac{1}{8} + \tfrac{1}{8} = \tfrac{1}{4}.$$

$$\sigma_X = \tfrac{1}{2}.$$

roll dice

1	2	3	4	5	6
$1/6$	. . .				$1/6$

$$\mu_X = 1 \cdot \tfrac{1}{6} + 2 \tfrac{1}{6} + \cdots + 6 \tfrac{1}{6} = \tfrac{21}{6} = 3\tfrac{1}{2}.$$

$$\sigma_X = (1 - 3.5)^2 \tfrac{1}{6} + (2 - 3.5)^2 \tfrac{1}{6} + \cdots + (6 - 3.5)^2 \tfrac{1}{6}$$

Rules for variance and standard deviation

① $\sigma_{a+bX}^2 = b^2 \sigma_X^2$

② if X and Y are independent then

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2.$$

$$\sigma_{X-Y}^2 = \sigma_X^2 + \sigma_Y^2.$$

③ if X and Y have correlation ρ then

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y.$$

$$\sigma_{X-Y}^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y.$$

Remark note: standard deviations don't add!

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}.$$