In general.

value of X	x_1	x_2	$\cdots$
probability of X	p_1	p_2	$\cdots$

$E(x) = \sum_i x_i p_i$.

Remark can think of tossing coin n times as taking a sample of size n.

Q: what is the expected sample mean?

Law of large numbers

Consider a population with (finite) mean μ.

Start choosing independent random observations $x_1, x_2, \ldots$

Let $\bar{x}_n$ be the sample mean of the first n samples.

then $\bar{x}_n \to \mu$ as $n \to \infty$.

Q: how fast? A good question: depends on the variability of the population.

Use coin toss data from class.

Rules for means Let X and Y be random variables with mean μ_X and μ_Y. Then:

① $\mu_{a+bX} = a + b\mu_X$

② $\mu_{X+Y} = \mu_X + \mu_Y$ (even if X, Y not independent!)

③ $\mu_{X-Y} = \mu_X - \mu_Y$.

Example

# cause in fall	1	2	3	4	5	6
prob.	0.05	0.05	0.13	0.22	0.36	0.15

$\mu_X = 4.28$

# cause in spring	1	2	3	4	5	6
prob	0.06	0.08	0.15	0.25	0.34	0.12

$\mu_Y = 4.09$

$\mu_{X+Y} = 4.28 + 4.09 = 8.37$.