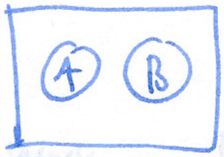
check $0.57 + 0.17 + 0.14 + 0.12 = 1$.

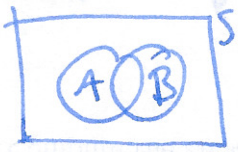
simplest case : finite probability models : set of events is finite, assign numbers to each event that add up to 1.

e.g. coin toss, roll dice.

Venn diagrams



$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$



$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.

Independent events if A, B independent then $\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$.

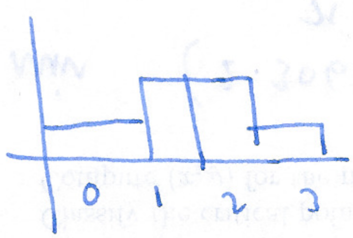
§4.3 Random variables

A random variable is a function from a probability space to numbers.

Example ① probability space : toss a coin. $X: H \longrightarrow 1$, $T \longrightarrow 0$.

② toss a coin 3 times : X: HTHH $\longrightarrow$ # heads etc

HHH	3
HHT	2
HTH	2
HTT	1
THH	2
THT	1
TTH	1
TTT	0



can ask : e.g. $\mathbb{P}$(get 2 heads)

mean # of heads

$\mathbb{E}$(# of heads)

$\mathbb{E}(X)$.

X has a prob dist : $\mathbb{P}(X = n)$

0	1	2	3
1/8	3/8	3/8	1/8

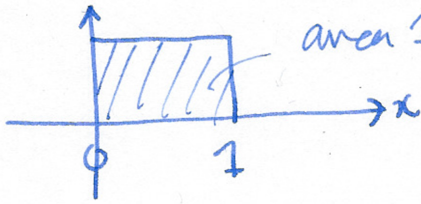
← less info than probability space!

discrete vs. continuous.

Do 4 coin tosses, or coin toss and odd dice. | do independence for toss coin twice.

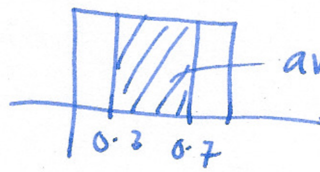
Continuous random variables

choose a real number between 0 and 1 uniformly at random.

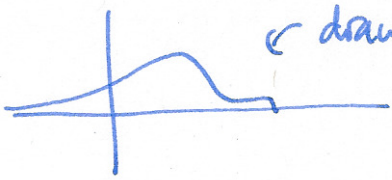


model: area corresponds to probability

$\mathbb{P}(0.3 < x < 0.7)$ is

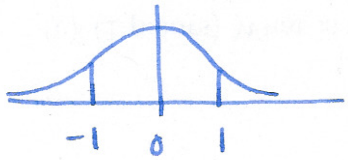


Example:



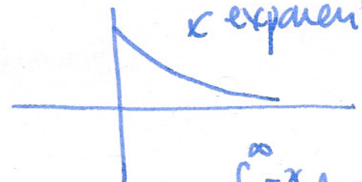
draw any graph with total area 1.

Example $N(0,1)$



or $N(\mu,\sigma)$.

Example



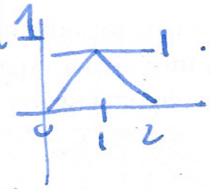
exponential dist. e^{-x}.

$$\int_0^\infty e^{-x}\,dx = \lim_{r\to\infty}\left[-e^{-x}\right]_0^r$$

$$= \lim_{r\to\infty} -e^{-r}+1 = 1.$$

so this is a probability distribution.

Example. what about picking two random numbers X_1, X_2 uniformly between 0 and 1. what is the probability distribution for X_1+X_2?

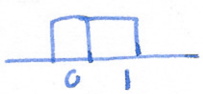


§4.4 Means and variances of random variables

discrete random variables: example: toss coin X. H win \$1. T win \$0.

$\mathbb{P}(X=1) = 1/2$

$\mathbb{P}(X=0) = 1/2$



$\mathbb{E}(X)$ = (average)mean value of $X = \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0 = \frac{1}{2} = \sum_{n\in\mathbb{Z}} n\mathbb{P}(X=n)$.

example: toss coin twice: HH win \$2, HT \$1, TT 0.

$\mathbb{P}(X=0) = 1/4$ $\mathbb{P}(X=1) = 1/2$ $\mathbb{P}(X=2) = 1/4$ $\mathbb{E}(X) = 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} = 1$.