§4.1 Randomness

Example: tossing a coin.

mathematical model: set of outcomes $\{H, T\}$

assign probabilities to subsets: $\mathbb{P}(\phi) = 0$

$\mathbb{P}(\{H\}) = \frac{1}{2}$ $\mathbb{P}(\{T\}) = 1/2$ $\mathbb{P}(\{H, T\}) = 1$

Independent sequence of coin tosses: mathematical model:

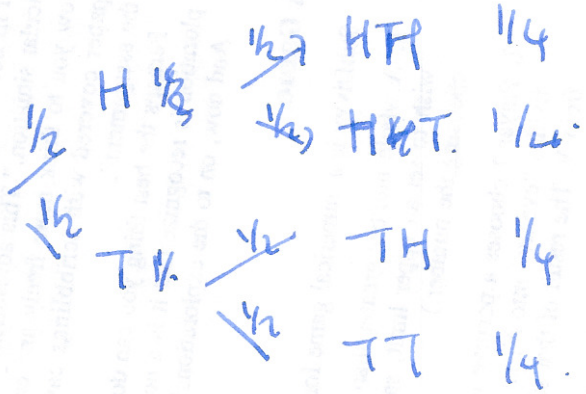
e.g. 2 coin tosses: $\{HH, HT, TH, TT\}$. (4 possibilities)

3 coin tosses: $\{HHH, HHT, HTH, \dots\}$ (8 possibilities).

independence: each coin toss doesn't influence the others

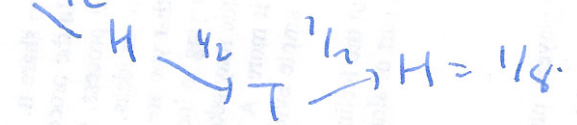
①	$\mathbb{P}$
H	1/2
T	1/2

②	
H	1/2
T	1/2



similarly

①	②	③
H 1/2	H 1/2	H 1/2
T 1/2	T 1/2	T 1/2



Q: what happen if you toss the coin lots of times?

Facts • $\#H - \#T \sim \sqrt{n}$.

• $\frac{\#H - \#T}{n} \sim \frac{1}{\sqrt{n}} \to 0$

so for large n, proportion of heads $\to \frac{1}{2}$ as $n \to \infty$. as $n \to \infty$

§4.2 Probability models

Sample space: set of events. e.g. roll a dice $\{1,2,3,4,5,6\}$.

event: subset of the sample space, e.g. get an even number $\{2,4,6\}$.

probability: assigns numbers to subsets. e.g. $\mathbb{P}(\{2,4,6\}) = 1/2$.

Example roll one dice $\{1,2,3,4,5,6\}$ $\mathbb{P}(\{1\}) = 1/6$ $\mathbb{P}(\{2\}) = 1/6$ etc.

roll two dice (independently): $\{(1,1),(1,2),(1,3)\dots,(1,6)$ ← 36 outcomes.
$(2,1),(2,2)\dots$
$\vdots$
$\{6,6\}\}$

$1/6 \cdot 1/6 = 1/36$

all with equal probability $1/36$.

Q. $\mathbb{P}(\text{roll } (6,6)) = 1/36$

$\mathbb{P}(\text{get numbers which sum to } 7) = \mathbb{P}(\{(1,6),(2,5),\dots(6,1)\}) = \frac{6}{36} = \frac{1}{6}$.

Probability rules 1. $0 \le \mathbb{P}(\text{event}) \le 1$

2. $\mathbb{P}(\text{something happens}) = 1$

3. If A, B are events which are disjoint (Ⓐ Ⓑ) then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$.

Example: roll 2 or 5 on one dice.

4. $\mathbb{P}(A \text{ does not occur}) = 1 - \mathbb{P}(A \text{ occurs})$.

Example: $\mathbb{P}(\text{not } 3) = 1 - 1/6 = 5/6$.

Example: a model for one cause: age of students.

Age group:	18 to 23	24 to 29	30 to 39	40 or over
$\mathbb{P}$.	0.57	0.17	0.14	0.12.