

Math 214 Statistics Spring 19 Sample Midterm 2

1. A researcher chooses 200 New Yorkers from the ages of 20–29 and polls them about the number of romantic partners they've had in the last year. She finds the polled individuals have had an average of 1.5 romantic partners. Assume that the standard deviation for this quantity for the population is 3, and assume that the sampling distribution of the sample mean is approximately normal.
 - (a) Compute a confidence interval with level 95% for the mean number of romantic partners for the population.
 - (b) Compute a confidence interval with level 88% for the mean number of romantic partners for the population.
2. A researcher conducts an experiment to see if a certain drug has an effect on pulse (number of heartbeats per minute). Test subjects are recruited from the general population, and their pulse is recorded. Then, they take the medication for a month, and their pulse is recorded again. Let μ be the mean value of (pulse after medication) - (pulse before medication) if all members of the general population were given the drug. Let $\bar{x}$ be the mean difference between the two pulses among the actual test subjects, of whom there are 64.
 - (a) State a null and alternative hypothesis for a test of significance for this experiment (either in words or in symbols is fine).
 - (b) Assume that the standard deviation for the difference of pulses in the population is 12, and assume that the sampling distribution of the sample mean is approximately normal. Suppose that $\bar{x} = 2$. What is the p-value for your test?
 - (c) What is your final decision regarding the hypotheses if you use a significance level of 0.05?
3. A team of agricultural researchers believes that tomato plants produce more fruit in September than in August and would like to design a study to test this. They propose studying the number of fruit produced in September minus the number of fruit produced in August for each plant. Let μ be mean value of this variable for all tomato plants. The researchers expect that the standard deviation for all tomato plants to be around 30.
 - (a) Write a null and alternative hypothesis for the researchers, either in words or in symbols.

- (b) The researchers propose planting 400 tomato plants and recording the number of fruit produced in September minus the number of fruit produced in August for each plant. Let $\bar{x}$ be the mean for this difference for the sample. For which values of $\bar{x}$ will the test reject the null hypothesis, and for which values will it fail to reject the null hypothesis? Use a 0.05 significance level for the test, and assume normality of the sampling distribution.
- (c) What is the power of the test to detect the alternative $\mu = 4.2$?

4. Complete the following sentences:

- (a) When conducting a hypothesis test, we _____ and then evaluate the test results to determine if there is enough evidence to _____.

Choose one answer:

- ☐ Assume that the null hypothesis is false; accept the null hypothesis
- ☐ Assume that the null hypothesis is true; reject the null hypothesis
- ☐ Assume that the alternative hypothesis is true; reject the null hypothesis
- ☐ Assume that the null hypothesis is true; reject the alternative hypothesis

- (b) If we decrease the significance level of our test from 0.05 to 0.01, it will make _____ and _____.

Choose one answer:

- ☐ type 1 errors more common; type 2 errors less common
- ☐ type 1 errors less common; type 2 errors more common
- ☐ the p-value smaller; the standard deviation larger
- ☐ the power of the test larger; the quality of research higher

- (c) The _____ is equal to the _____ divided by the square root of the number of samples.

Choose one answer:

- ☐ standard deviation of the sampling distribution of the sample mean; population standard deviation
- ☐ sample standard deviation; population standard deviation
- ☐ standard deviation of the sampling distribution of the sample mean; sample standard deviation

☐ sample mean; population mean

5. There are 100,000 people living in a certain neighborhood. A researcher would like to know the mean income in the neighborhood. In fact, its exactly \$74,341 per year, but the researcher doesnt know this.

The researcher would like to test whether the mean income in the neighborhood is greater than \$60,000. She chooses a sample of 1000 randomly selected inhabitants of the neighborhood, determines their income, and computes that the mean income in the sample is \$69,421. She then uses this to compute a 95% confidence interval for the population parameter. She also performs a test of significance with null hypothesis that the mean income for the neighborhood is exactly \$60,000 and alternative hypothesis that its greater than \$60,000.

She computes the p-value for the test and the 95% confidence interval for the parameter.

- (a) What is the value of μ , the population mean of income?
- (b) What is the value of $\bar{x}$, the sample mean of income?
- (c) Which of the following statements is true? Choose one:
 - ☐ Out of every 100 individuals in the neighborhood, the income of about 95 will fall in the confidence interval.
 - ☐ If 100 researchers conduct their own samples and carry out the same tests, about 95 of their confidence intervals will contain \$74,341.
 - ☐ Out of every 100 individuals in the neighborhood, the income of about 95 will be greater than \$60,000.
 - ☐ If 100 researchers conduct their own samples and carry out the same tests, about 95 of their confidence intervals will contain \$69,421.
- (d) Suppose that the mean income of people in the poll is \$70,421 instead of \$69,421. Then the p-value will be:
 - ☐ larger
 - ☐ smaller
- (e) Suppose that the standard deviation of the sampling distribution of the sample mean is decreased, but the same sample mean stays the same. Then the p-value will be:
 - ☐ larger
 - ☐ smaller

6. You do a test of significance with hypotheses:

- $H_0 : \mu = 20$
- $H_a : \mu \neq 20$

You compute

$$z = \frac{\bar{x} - 20}{\sigma/\sqrt{n}}$$

where $\bar{x}$ is the sample mean, σ is the standard deviation for the population (whose value you know), and n is the sample size. You find that $z = 1.82$.

- (a) Was $\bar{x}$ larger or smaller than 20?
- ☐ larger
 - ☐ smaller
 - ☐ not enough information given
- (b) What is the p-value for your test?
- (c) Suppose everything is the same, except that the alternative hypothesis is that $\mu > 20$. What is the p-value?
- (d) Suppose everything is the same, except that the alternative hypothesis is that $\mu < 20$. What is the p-value?
7. New York is known as the city that never sleeps. A random sample of 25 New Yorkers were asked how much sleep they get per night. Statistical summaries of these data are shown below. Do these data provide strong evidence that New Yorkers sleep less than 8 hours a night on average?

n	$\bar{x}$	s	min	max
25	7.73	0.77	6.17	9.78

- (a) Write the hypotheses in symbols and in words.
- (b) Check conditions, then calculate the test statistic, t , and the associated degrees of freedom.
- (c) Find and interpret the p-value in this context. Drawing a picture may be helpful.

- (d) What is the conclusion of the hypothesis test?
 - (e) If you were to construct a 90% confidence interval that corresponded to this hypothesis test, would you expect 8 hours to be in the interval?
 - (f) Construct the 90% confidence interval.
8. The table provides summary statistics on highway fuel economy of cars manufactured in 2012. Use these statistics to calculate a 98% confidence interval for the difference between average highway mileage of manual and automatic cars, and interpret this interval in the context of the data.

	Highway MPG	
	Automatic	Manual
Mean	22.92	27.88
SD	5.29	5.01
n	26	26

9. We are interested in estimating the proportion of graduates at a mid-sized university who found a job within one year of completing their undergraduate degree. Suppose we conduct a survey and find out that 348 of the 400 randomly sampled graduates found jobs. The graduating class under consideration included over 4500 students.
- (a) Describe the population parameter of interest. What is the value of the point estimate of this parameter?
 - (b) Check if the conditions for constructing a confidence interval based on these data are met.
 - (c) Calculate a 95% confidence interval for the proportion of graduates who found a job within one year of completing their undergraduate degree at this university, and interpret it in the context of the data.
 - (d) What does “95% confidence” mean?
 - (e) Now calculate a 99% confidence interval for the same parameter and interpret it in the context of the data.
 - (f) Compare the widths of the 95% and 99% confidence intervals. Which one is wider? Explain.
10. According to a report on sleep deprivation by the Centers for Disease Control and Prevention, the proportion of California residents who reported insufficient rest or sleep during each of the preceding 30 days is 8.0%, while this proportion

is 8.8% for Oregon residents. These data are based on simple random samples of 11,545 California and 4,691 Oregon residents.

- (a) Calculate a 95% confidence interval for the difference between the proportions of Californians and Oregonians who are sleep deprived and interpret it in context of the data.
- (b) Does this information provide significant evidence that Oregonians are more sleep deprived than Californians? Set up and apply a hypothesis test.

Formulas

μ	population mean
σ	population standard deviation
n	sample size
$\bar{x}$	sample mean
s	sample standard deviation (standard error)
p	population proportion
$\hat{p}$	sample proportion

The sample mean $\bar{x}$ of a normal distribution $N(\mu, \sigma)$ has distribution $N(\mu, \sigma/\sqrt{n})$.

The sample mean of any distribution with mean μ and standard deviation σ has distribution approximately $N(\mu, \sigma/\sqrt{n})$, for n sufficiently large.

	Confidence interval	Test statistic	Distribution
mean, known σ	$\bar{x} \pm z_* \sigma / \sqrt{n}$	$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$	$N(0, 1)$
mean, unknown σ	$\bar{x} \pm t_* s / \sqrt{n}$	$\frac{\bar{x} - \mu}{s / \sqrt{n}}$	t-dist, df = $n - 1$
difference between two means	$\bar{x}_1 - \bar{x}_2 \pm t_* \sqrt{s_1^2/n_1 + s_2^2/n_2}$	$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$	t-dist, df = $\min\{n_1, n_2\} - 1$
proportion	$\hat{p} \pm z_* \sqrt{\hat{p}(1 - \hat{p})/n}$	$\frac{\hat{p} - p}{\sqrt{p(1 - p)/n}}$	$N(0, 1)$
difference between two proportions	$\hat{p}_1 - \hat{p}_2 \pm z_* \sqrt{\hat{p}(1 - \hat{p})(1/n_1 + 1/n_2)}$, where $\hat{p} = (\hat{p}_1 n_1 + \hat{p}_2 n_2) / (n_1 + n_2)$	$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})(1/n_1 + 1/n_2)}}$	$N(0, 1)$

$$\chi^2 \text{ statistic: } \chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Correlation coefficient: $r = \frac{1}{n-2} \sum \frac{(x_i - \bar{x})}{s_x} \frac{(y_i - \bar{y})}{s_y}$

Regression line: $\hat{y} = b_1x + b_0$, where $b_1 = r \frac{s_y}{s_x}$ and $b_0 = \bar{y} - b_1\bar{x}$

Residuals: $e_i = y_i - \hat{y}_i$

Confidence intervals for population regression parameters β_0 and β_1 .

$$b_1 \pm t_* SE_{b_1} \qquad b_0 \pm t_* SE_{b_0}$$

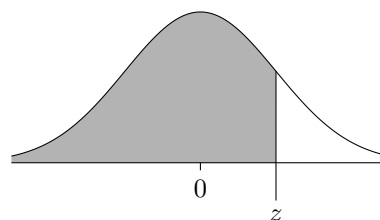
$$SE_{b_1} = \frac{s}{\sqrt{\sum (x_i - \bar{x})^2}}$$

$$SE_{b_0} = s \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\sum (x_i - \bar{x})^2}}$$

$$s^2 = \frac{\sum e_i^2}{n-2}$$

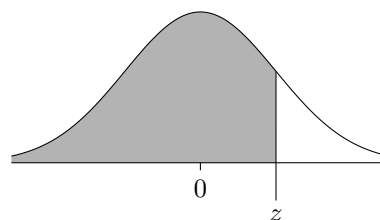
The hypothesis test for $H_0: \beta_1 = 0$ is based on the t -statistic $t = \frac{b_1}{SE_{b_1}}$ and the t -distributions with $(n-2)$ -degrees of freedom.

Standard normal distribution table



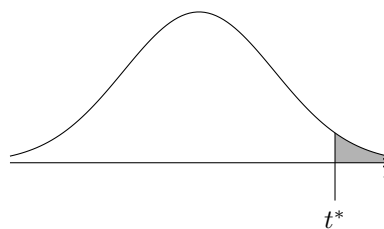
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002

Standard normal distribution table

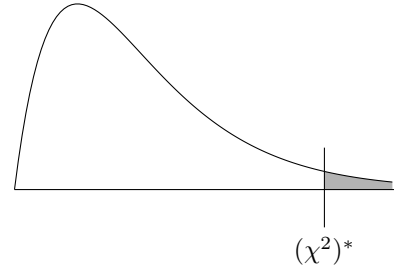


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

t-distribution critical values



df	Upper tail probability p											
	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0200	0.0100	0.0050	0.0025	0.0010	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291

χ^2 -distribution critical values

df	Tail probability p (one-sided)											
	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0200	0.0100	0.0050	0.0025	0.0010	0.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.52	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.69
80	88.13	90.41	93.11	96.58	101.88	106.63	108.07	112.33	116.32	120.10	124.84	128.26
100	109.14	111.67	114.66	118.50	124.34	129.56	131.14	135.81	140.17	144.29	149.45	153.17