Math 214 Statistics Spring 19 Sample Final

0. (a) You roll two six-sided dice. What is the probability that

   i. Both numbers are odd?

   ii. The sum of the numbers is odd?

   (b) The average systolic blood pressure of 19–24 year olds is 120 mm Hg. You take a sample of 24 CSI students. The average blood pressure is 128 with a standard deviation of 12.3.

   i. Describe a possible hypothesis test to test if CSI students have average blood pressure or not.

   ii. What are the critical vlaues for your test?

   iii. Apply your test, and compute the p-value.

   iv. Find a 95% confidence interval for the average blood pressure of CSI students.

   v. What is the power of your test to detect the alternative hypothesis $H_a : \mu = 130$. (To simplify this calculation you may aprroximate the t-distributions with normal distributions.)

Here is a list of tests that are possible answers in Question 1–4:

1. One-sample t-test
2. Paired t-test
3. Two-sample t-test
4. One-sample proportion test
5. Two-sample proportion test
6. Chi-square test of goodness of fit
7. Chi-square test of independence
8. Linear regression

Question 1

A sociologist collects a sample of 100 individuals sampled randomly in the state of Texas. She has polled them at the beginning of 2017 and again at the beginning of 2018, asking them to rate on a scale of 0–10 whether the country is on the right track or the wrong track (0=completely wrong track, 10=completely right track). She is interested in determining if people are more pessimistic in 2018 than 2017.

- **What test would you use to investigate this question?** Just give the number.

- **Write null and alternative hypotheses to investigate this question.**

Question 2

A researcher is interested in determining if the rate of childhood asthma in a midwestern town decreased after a coal power plant was closed down. Before the closure, he obtained a random sample of 75 five-year-olds and determined how many of them had asthma. After the closure, he obtained a new sample of 75 five-year-olds and again determined how many of them had asthma.

- **What test would you use to investigate this question?** Just give the number.

- **Write null and alternative hypotheses to investigate this question.**

Question 3

You would like to know if births are equally likely to occur on each of the seven days of the week. You obtain a random sample of the births in 2017 and determine the day of the week each birth occurred.

- **What test would you use to investigate this question?** Just give the number.

- **Write null and alternative hypotheses to investigate this question.**

Question 4

A health study tracks 500 randomly sampled individuals in the United States. The dataset categorizes individuals into three categories with regard to fish consumption, `<1 portion/week`, `1-2 portions/week`, and `>2 portions/week`. They also track whether each individual suffers from heart disease, recording the response as `yes` or `no`. You are interested in whether the proportions of people suffering from heart disease differ among the three categories of fish consumption.

- **What test would you use to investigate this question?** Just give the number.

- **Write null and alternative hypotheses to investigate this question.**

Question 5

A study investigated whether low to moderate levels of alcohol use by women during pregnancy was associated with behavioral problems in their children later in their lives.[1] Women were interviewed about their drinking habits 18 weeks into their pregnancy. Fifteen years later, they were interviewed again about whether their child showed any behavioral problems.

In the dataset for the study, each observation represents one pregnancy. One variable records how much the mother reported drinking in the pregnancy, `"no drinking"`, `"<=1 drink/week"`, or `"2-6 drinks/week"`. Another variable reports whether the child at age 14 displays behavioral problems, coded either as `"no behavioral problems"` or `"has behavioral problems"`. The researchers made a two-way table from the study data:

```r
twoway.table
```

```
##
##                    has behavioral problems no behavioral problems
##   no drinking                          145                    752
##   <=1 drink/week                        56                    388
##   2-6 drinks/week                       39                    299
```

Then, they ran the following test:

```r
chisq.test(twoway.table)
```

```
##
##  Pearson's Chi-squared test
##
## data:  twoway.table
## X-squared = 5.6826, df = 2, p-value = 0.05835
```

- **What were the null and alternative hypotheses tested?**

- **At a 5% significance level, what is the result of the test?**

[1] M. Robinson et al., "Low-moderate prenatal alcohol exposure and risk to child behavioural development: a prospective cohort study." BJOG. 2010 Aug; 117(9):1139-50.

Question 6

Researchers compare salmon size in the Columbia River and the Lewis River. They gather a random sample of salmon in each river and weigh them, storing the data in pounds as `weights.columbia` and `weights.lewis`. They perform the following two-sample t-test:

```r
t.test(weights.columbia, weights.lewis, mu=0, alternative="greater")
```

```
##
##  Welch Two Sample t-test
##
## data:  weights.columbia and weights.lewis
## t = 0.18242, df = 67.091, p-value = 0.4279
## alternative hypothesis: true difference in means is greater than 0
## 95 percent confidence interval:
##  -0.6309684        Inf
## sample estimates:
## mean of x mean of y
##  11.36711  11.28962
```

- **What were the null and alternative hypotheses tested?**

- **What was the mean weight for the Columbia River salmon in the sample?**

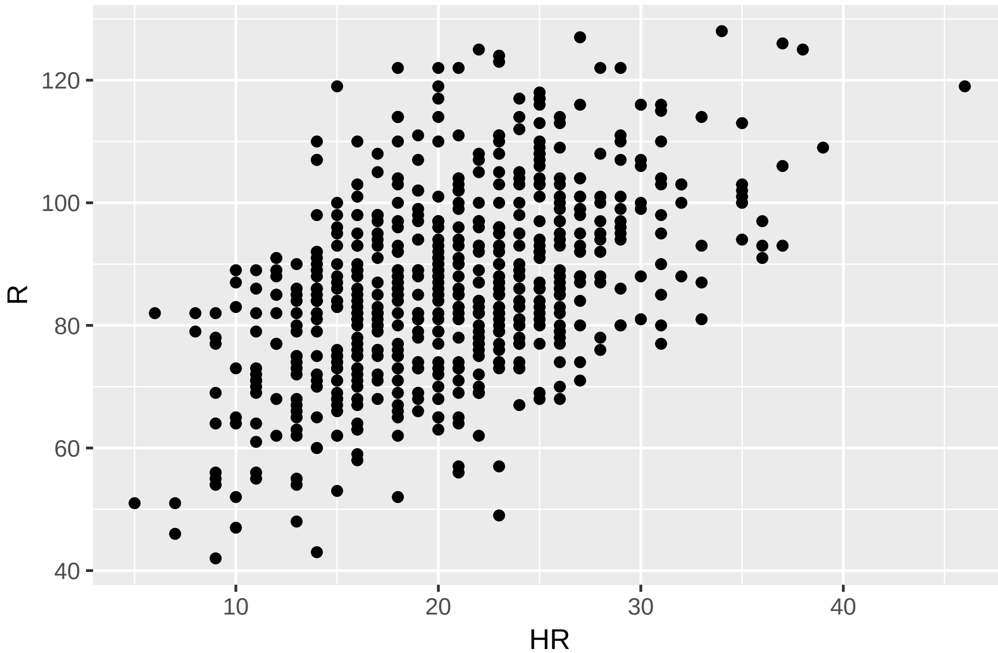
- **What was the mean weight for the Lewis River salmon in the sample?**

- **What is the result of the test, at a 5% significance level?**

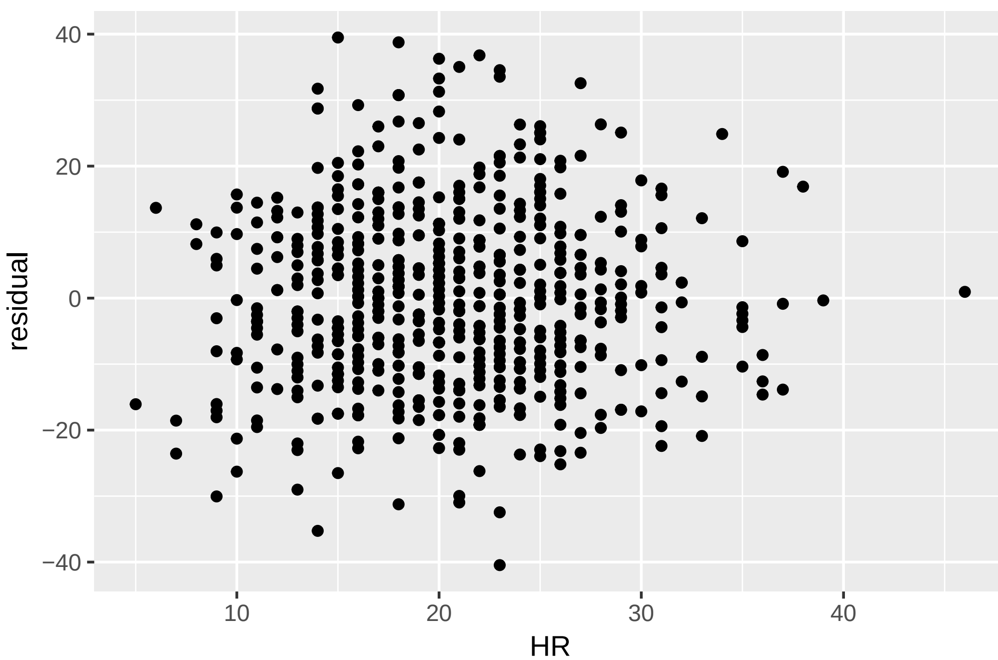
Question 7

You have a dataset `baseball` that contains information about baseball pitchers from 2010 to 2017. Each observation gives information about the performance of a baseball player over one year. The dataset includes a variable `HR` giving the number of home runs the player allowed, another variable `R` giving the total number of runs the player allowed. You would like to fit a simple linear regression model with `HR` as the explanatory variable and `R` as the response variable.

Here is a scatterplot of the dataset:

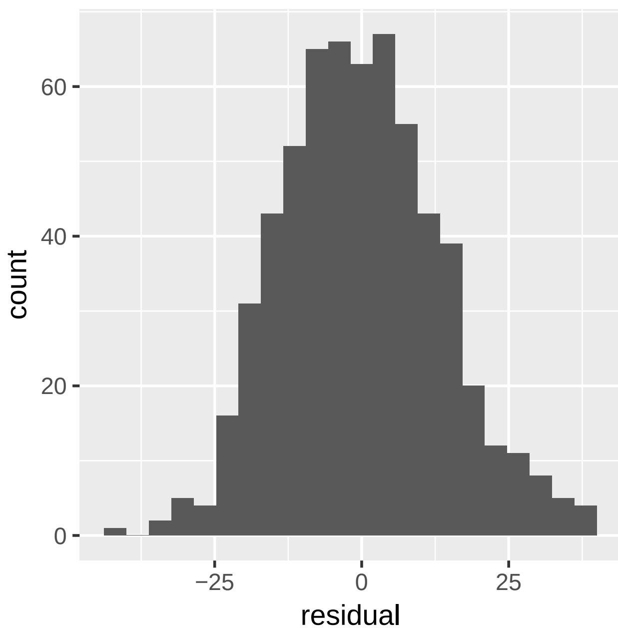


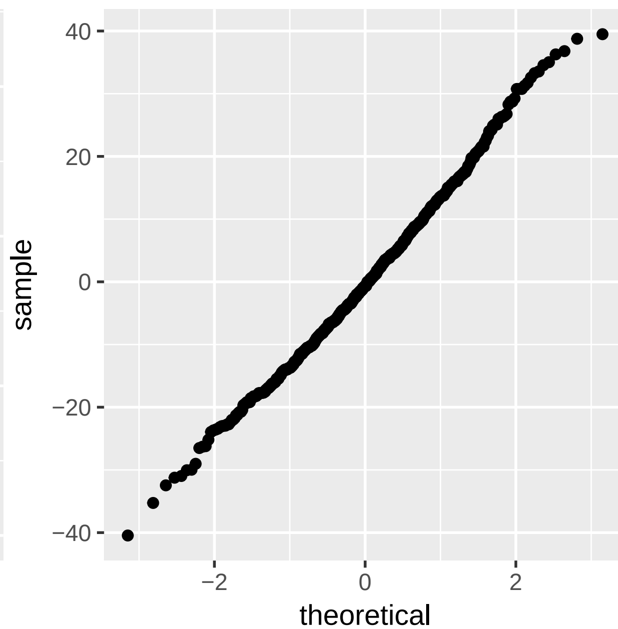
Here is a scatterplot showing the residuals for the regression:



Here's a histogram and Q-Q plot of the residuals:

```
ggplot(baseball, aes(x=residual)) + geom_histogram(bins=22)
ggplot(baseball, aes(sample=residual)) + stat_qq()
```





- **List the conditions for simple linear regression modeling. Judge if they're satisfied.** Ignore the *simple random sample* condition.

Question 8

With the data from the previous problem, you run the following commands in R. Regardless of your answer to the previous problem, assume now that inference for the linear regression model is valid.

```
results <- lm(R~HR, data=baseball)
summary(results)
```

```
##
## Call:
## lm(formula = R ~ HR, data = baseball)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -40.456  -9.700  -0.713   8.800  39.490
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 60.86080    1.92228   31.66   <2e-16 ***
## HR           1.24328    0.08967   13.87   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.42 on 610 degrees of freedom
## Multiple R-squared:  0.2396, Adjusted R-squared:  0.2384
## F-statistic: 192.3 on 1 and 610 DF,  p-value: < 2.2e-16
```

```
xvals <- data.frame(HR=c(20))
predict(results, xvals, interval="confidence", level=.95)
```

```
##        fit      lwr      upr
## 1 85.72644 84.65639 86.79649
```

```
predict(results, xvals, interval="prediction", level=.95)
```

```
##        fit      lwr      upr
## 1 85.72644 59.34863 112.1042
```

- **Circle *TRUE* or *FALSE*:**
  - There is strong evidence that in the linear model for the population, the mean of the `R` variable depends on the `HR` variable.

    TRUE **FALSE**

  - Because the prediction intervals are wider than the confidence intervals, there is a nonlinear relationship between the two variables.

    TRUE **FALSE**

  - According to the model, 23.96% of the variance in the `R` values is explained by the regression line, and the rest of the variance is explained by the deviations from the regression line.

    TRUE **FALSE**

  - One would expect an observation whose value for the `HR` variable is 20 to have a value for the `R` variable between 84.65639 and 86.79649.

    TRUE **FALSE**

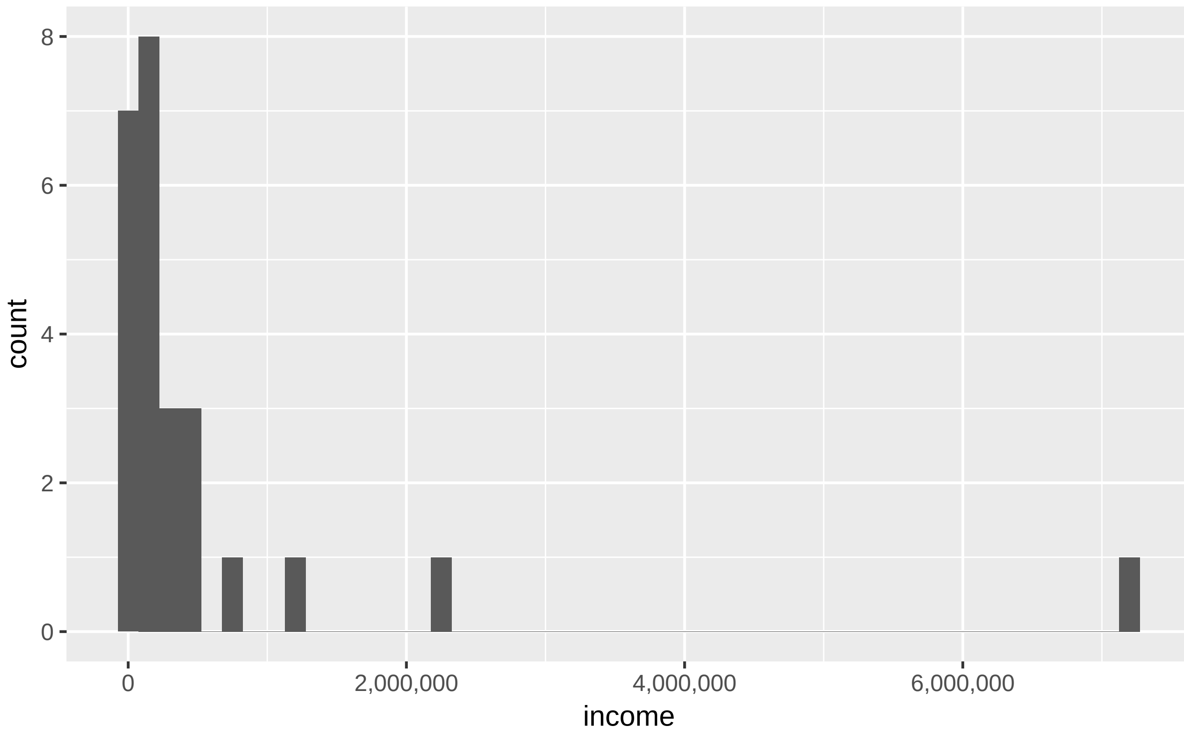
Question 9

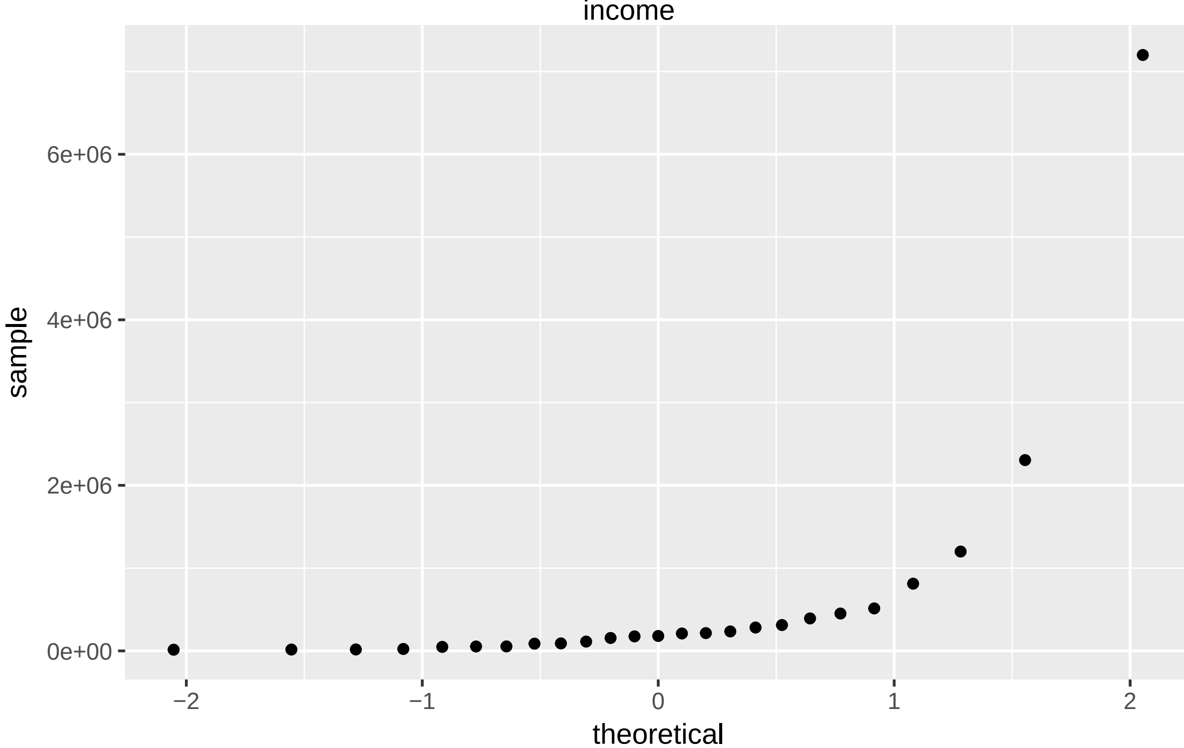
You run an experiment to compare if a new method of knee surgery has a lower rate of complications than the standard one. Patients are assigned at random to receive one of the surgeries. In the standard surgery group, 16 out of 100 patients experience complications. In the new surgery group, 8 out of 100 patients experience complications. You want to do a two-sample proportion test with null hypothesis that the complication rates in the two groups are equal and alternative hypothesis that they are not equal.

- **List the conditions for the test of significance to provide valid results. Judge if they're satisfied.** (You can assume that the patients represent a simple random sample from the population of knee surgery patients.)

Question 10

The College of Staten Island plans a study to determine the average income of their alumni in different majors. They take a random sample of their graduates and determine their incomes. For the subpopulation of mathematics majors, the sample has 25 observations. Here is a histogram and Q-Q plot of the sample data:





- **List the conditions for valid inference of the mean income of mathematics majors using the t-distribution. Give your judgment as to whether they are satisfied.**

Question 11

A city government chooses 500 real estate transactions sampled randomly from all real estate transactions during the year and investigates them for signs of financial fraud. They find that 23 out of the 500 transactions are fraudulent.

- **Give a 95% confidence interval for the proportion of real estate transactions that are fraudulent.**

- The city would like to improve its estimate of the proportion of real estate transactions that are fradulent to have a 1% margin of error for the 95% confidence interval. **How large should their sample size be?** Use the sample proportion from the previous problem as the best guess at the true proportion in your calculations.

Question 12

A medical researcher conducts a very small experiment to see if counseling reduces cigarette smoking. Smokers are split at random into one group that receives counseling and one that doesn't. The number of cigarettes smoked per day is recorded for each patient. The researcher does a two-sample t-test with alternative hypothesis that the counseling group smokes fewer cigarettes on average than the non-counseling group. She finds a p-value of .18.

- **Circle *TRUE* or *FALSE*:**
  - The researcher should reject the null hypothesis.
    TRUE **FALSE**
  - There is evidence that counselling reduces cigarette consumption, but only by a small amount.
    TRUE **FALSE**
  - The study establishes that the counseling had no effect on cigarette consumption.
    TRUE **FALSE**
  - The study did not find evidence that counseling had an effect on cigarette consumption, but it's possible that the sample size was too small to detect this effect.
    TRUE **FALSE**
  - Regardless of the results of the study, its design only allows it to establish association, not causation.
    TRUE **FALSE**

Question 13

- **Respond *TRUE* or *FALSE* to the following statements.**
  - If an experiment design has very low power, it probably will not reject the null hypothesis even if the alternative hypothesis is true.
    TRUE **FALSE**
  - If an experiment design has very low power and the alternative hypothesis is actually true, a type 2 error might occur.
    TRUE **FALSE**
  - If the 95% confidence interval for the mean household income in Manhattan is from $82,345 to $103,284, then approximately 95% of households in Manhattan have household incomes between $82,345 and $103,284.
    TRUE **FALSE**

Question 14

Suppose you perform a one-sample t-test with alternative hypothesis $\mu \neq 12$ and find that the sample mean $\bar{x}$ is 10 and the sample standard deviation s is 2.

- **Respond *TRUE*, *FALSE*, or *NOT ENOUGH INFO* to the following statements.**
  - If s were 1 instead of 2, the p-value for the test would be smaller.

    TRUE **FALSE** **NOT ENOUGH INFO**

  - If $\bar{x}$ were 9 instead of 10, the p-value for the test would be smaller.

    TRUE **FALSE** **NOT ENOUGH INFO**

  - The t-score for the test is positive.

    TRUE **FALSE** **NOT ENOUGH INFO**

Question 15

Suppose that a test of significance with significance level .05 has a p-value of .37.

- **Respond *TRUE* or *FALSE* to the following statements:**
  - According to the test, you should reject the null hypothesis.

    TRUE **FALSE**

  - There is a 37% chance that the null hypothesis is correct and a 63% chance that the alternative hypothesis is correct.

    TRUE **FALSE**

  - There is a 37% chance that the alternative hypothesis is correct and a 63% chance that the null hypothesis is correct.

    TRUE **FALSE**

  - The power of the test to detect the alternative hypothesis is 37%.

    TRUE **FALSE**

  - Assuming the truth of the null hypothesis, there is a 37% chance of observing behavior as favorable to the alternative hypothesis as what you observed.

    TRUE **FALSE**

  - Assuming the truth of the alternative hypothesis, there is a 37% chance of observing behavior as favorable to the null hypothesis as what you observed.

    TRUE **FALSE**

Formulas

μ	population mean
σ	population standard deviation
n	sample size
$\overline{x}$	sample mean
s	sample standard deviation (standard error)
p	population proportion
$\hat{p}$	sample proportion

The sample mean $\overline{x}$ of a normal distribution $N(\mu, \sigma)$ has distribution $N(\mu, \sigma/\sqrt{n})$.

The sample mean of any distribution with mean μ and standard deviation σ has distribution approximately $N(\mu, \sigma/\sqrt{n})$, for n sufficiently large.

	Confidence interval	Test statistic	Distribution
mean, known σ	$\overline{x} \pm z_* \sigma/\sqrt{n}$	$\dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$	$N(0, 1)$
mean, unknown σ	$\overline{x} \pm t_* s/\sqrt{n}$	$\dfrac{\overline{x} - \mu}{s/\sqrt{n}}$	t-dist, df $= n - 1$
difference between two means	$\overline{x_1} - \overline{x_2} \pm t_* \sqrt{s_1^2/n_1 + s_2^2/n_2}$	$\dfrac{\overline{x}_1 - \overline{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$	t-dist, df=min$\{n_1, n_2\} - 1$
proportion	$\hat{p} \pm z_* \sqrt{\hat{p}(1-\hat{p})/n}$	$\dfrac{\hat{p} - p}{\sqrt{p(1-p)/n}}$	$N(0, 1)$
difference between two proportions	$\hat{p}_1 - \hat{p}_2 \pm z_* \sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}$, where $\hat{p} = (\hat{p}_1 n_1 + \hat{p}_2 n_2)/(n_1 + n_2)$	$\dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}}$	$N(0, 1)$

χ^2 statistic: $\chi^2 = \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

Correlation coefficient: $r = \dfrac{1}{n-2} \sum \dfrac{(x_i - \overline{x})}{s_x} \dfrac{(y_i - \overline{y})}{s_y}$

Regression line: $\widehat{y} = b_1 x + b_0$, where $b_1 = r\dfrac{s_y}{s_x}$ and $b_0 = \overline{y} - b_1 x$

Residuals: $e_i = y_i - \widehat{y}_i$

Confidence intervals for population regression parameters β_0 and β_1.

$$b_1 \pm t_* SE_{b_1} \qquad\qquad b_0 \pm t_* SE_{b_0}$$

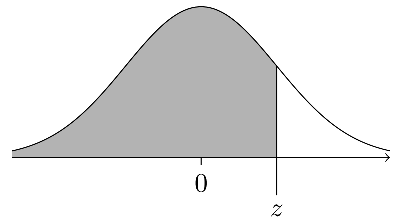
$$SE_{b_1} = \frac{s}{\sqrt{\sum (x_i - \overline{x})^2}}$$

$$SE_{b_0} = s\sqrt{\frac{1}{n} + \frac{\overline{x}^2}{\sum (x_i - \overline{x})^2}}$$

$$s^2 = \frac{\sum e_i^2}{n-2}$$

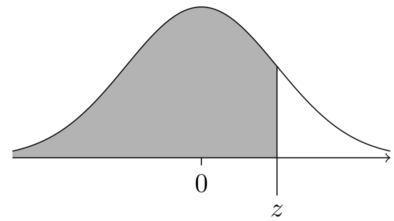
The hypothesis test for $H_0\colon \beta_1 = 0$ is based on the t-statistic $t = \dfrac{b_1}{SE_{b_1}}$ and the t-distributions with $(n-2)$-degrees of freedom.

Standard normal distribution table



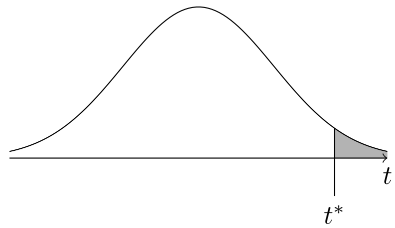
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002

Standard normal distribution table



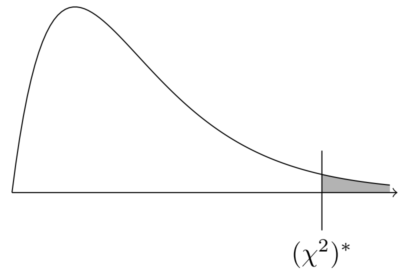
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

t-distribution critical values



	Upper tail probability p											
df	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0200	0.0100	0.0050	0.0025	0.0010	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291

χ^2-distribution critical values



	Tail probability p (one-sided)											
df	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0200	0.0100	0.0050	0.0025	0.0010	0.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.52	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.69
80	88.13	90.41	93.11	96.58	101.88	106.63	108.07	112.33	116.32	120.10	124.84	128.26
100	109.14	111.67	114.66	118.50	124.34	129.56	131.14	135.81	140.17	144.29	149.45	153.17