Class 3 §1.3 Proof: an introduction

Q: why prove things? A: certainty, correcting mistakes, gaining insight, generalization, computational guidance.

A proof is a specia kind of explanation, usually precise and formal. Level of detail depends on context.

Q: why is the sum of two even numbers even?

Example Thm 1.1 If x and y are even integers, then $x+y$ is an even integer.

Proof 1) begin by assuming x and y are even integers.

2) what does it mean to be even? A multiple of 2, i.e. $x = 2m$, $y = 2n$ for some integers m and n.

3) what does this say about $x+y$? $x+y = 2m+2n = \underbrace{2(m+n)}_{\text{even}}$. ◻
sum of two integers is an integer, so $x+y$ is even ◻.

Observations

- statement of theorem: a precise statement that can be proved, with specific assumptions. Typically: if ..... then ...... ← conditional sentence.
  hypotheses. conclusion

step 1 • assume hypothesis. (and then derive conclusion).

- x, y known as variables: what do they mean?
  where do they come from?
  what are the rules for working with them?

step 2 interpret the assumptions
in this case words ⇝ algebraic form.
– directly uses the definition in this case (common in basic proofs)

remark: x even means x can be divided by 2
means $x = 2m$ m integer ← this turns out to be more useful.

step 3 do work: in this case use distributive property
$2m+2n = 2(m+n)$

- want intuition for this step, can be long and complicated. ③

EXHD 1: if x is an even integer and y is a multiple of x, then y is even.

Conditional sentences If .... then ....
hypothesis conclusion

Examples If x,y odd integers then xy odd

If two sides of a triangle equal length then two angles equal

If x,y real numbers, then $|x+y| \le |x|+|y|$

If a function is differentiable then it is continuous.

If P then Q.

$$P \Rightarrow Q.$$

doesn't matter what happens if P is false!

Warning: lots of different ways to say this: e.g.: $a+b$ is even whenever a and b are even.

Variables Thm 1.1: hyp says x,y are even integers.

Q: which ones? A: don't know.

How not to prove Thm 1.1: suppose $x=8$ $y=12$ then $8+12=20$ even.

need to deal with all even integers, so x is some even integer, we call this a variable.

Open sentences: a sentence containing a variable

Examples		variables	typ
	$\cdot\ a>b$	a,b	numbers
	$\cdot\ S \subseteq \mathbb{Z}$	S	subset of $\mathbb{Z}$ set
	$\cdot\ A \cap B = \emptyset$	A,B	sets
	$\cdot\ y^2-5y+6=0$	y	number
	$\cdot\ z \in C \cup D$	z,C,D	z element C,D sets

substitution : $\overset{x}{\cancel{9}}$ is even $\overset{x}{\cancel{4}}$ variable x number. ~~3~~ ~~4~~ 4

x is even

4 is even T
−7 is even F
0 is even T

truth values.

the collection of objects which make a given open sentence true is called its truth set.

x is even : need to know what x can refer to
in this case $x \in \mathbb{Z}$
truth set is $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$

Example If $\underbrace{x \text{ is prime, } x>2}_{\text{hyp}}$ then $\underbrace{x \text{ is odd}}_{\text{conclusion}}$

open sentences

truth set: $\{3, 5, 7, 11, \ldots\}$ $\{\ldots, -3, -1, 1, 3, 5, \ldots\}$

When is a conditional statement true?

when truth set for hypoth is a subset of truth set for conclusion

When is a conditional statement false?

" not a subset. "

Counterexamples : Example: if x is a positive integer then x is even

$\{1, 2, 3, 4, \ldots\}$ $\{\ldots, -2, 0, 2, \ldots\}$

just need to find one example which doesn't work. i.e. 3.

Note : a conditional sentence is false if values can be found for the variables which make the hyp true and the conclusion false.

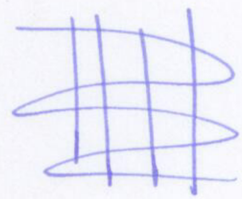
substitutions.

hyp	conclusion	
true	true	✓
false	true	✓
false	false	✓
true	false	X counterexample.

Definitions Example Defn An even integer is divisible by 2 (if and only if, iff)

not a definition: an integer which is divisible by 4 is even (not iff).

Motivation: proof for certainty
understanding



Q: how many pieces?