Math 505 Introduction to Proofs Spring 19 Sample Midterm 1

(1) Find three distinct elements for the truth sets of the following statements:
    (a) $x^2 + y^2 = 1$, where the universe is $\mathbb{R} \times \mathbb{R}$.
    (b) A is a subset of $\mathbb{Z}$ with exactly three elements.
    (c) A is an element of $\mathcal{P}(\mathbb{Z})$.
    (d) A is a subset of $\mathcal{P}(\mathbb{Z})$.

(2) Consider the statement:

    If n is an even number, then n is not a cube.

    Which, if any, of the following substitutions give a counter example.
    (a) $n = 27$
    (b) $n = 64$
    (c) $n = 16$

(3) Write out a careful proof of the fact that the square of an odd number is odd.

(4) What is $\mathcal{P}(\{1\})$? What is $\mathcal{P}(\mathcal{P}(\{1\}))$?

(5) If $A \cup B = A \cup C$ does this imply that $B = C$?

(6) State which of the following statements, are true, vacuously true, or false.
    (a) If $A \cap B \supseteq A$, then $A \subseteq B$.
    (b) If $\mathcal{P}(A) = \varnothing$, then $A = \varnothing$.
    (c) If $A \in B$ and $B \in C$, then $A \in C$.

(7) Suppose A and B are finite sets with $|A| = a$, $|B| = B$ and $|A \cap B| = c$. Find
    (a) $|A \setminus B|$
    (b) $|B \times (A \cap B)|$
    (c) $|\mathcal{P}(A \cap B)|$

(8) There is an island where all people either always lie, or always tell the truth.
    You meet three people, A, B and C.
    A says: At least two of us are truth tellers.
    B says: A is lying.
    C says: B is lying.
    What can you deduce?

(9) There is an island where all people either always lie, or always tell the truth. You meet D who says "I am lying." What can you deduce?