

Q9 This is a contradiction - statement inconsistent with assumption everyone tells the truth or lies.

- Q1 a)  $(1,0), (0,1), (-1,0)$ .  
 b)  $\{0,1,2\}$   
 c)  $\{1\}$  d)  $\{\emptyset, \{1\}, \mathbb{Z}\}$ .

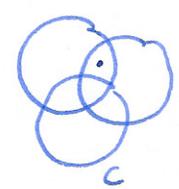
Q2 b) is a counterexample.

Q3 Thm: The square of an odd number is odd.

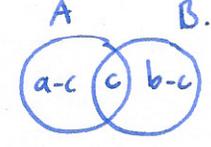
Proof Let  $n$  be an odd number, then  $n = 2a+1$  for some  $a \in \mathbb{Z}$ .

Then  $n^2 = (2a+1)^2 = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1$ . As  $a \in \mathbb{Z}$ ,  $2a^2 + 2a \in \mathbb{Z}$ , and so  $n^2$  is odd, as required.  $\square$

Q4  $P(\{1,3\}) = \{\emptyset, \{1,3\}\}$ .  $P(P(\{1,3\})) = \{\emptyset, \{\emptyset\}, \{\{1,3\}\}, \{\emptyset, \{1,3\}\}\}$ .

Q5 No.  Counterexample  $A=B=\{1,3\}$   $C=\emptyset$ .  
 $A \cup B = \{1,3\}$  -  $A \cup C = \{1,3\}$  but  $B \neq C$ .

Q6 a) true b) vacuously true c) false

Q7  a)  $a-c$  b)  $(b)(c) = bc$  c)  $2^c$

Q8

A	B	C	A's statement <sup>&gt;&gt;TT</sup> <del>is</del> consistent	A is lying consistent	B is lying consistent
T	T	T	T ✓	F x	
T	T	F	T ✓	F x	
T	F	T	T ✓	F ✓	T ✓
T	F	F	F ✗	T	
F	T	T	T ✗	F	
F	T	F	F ✓	T ✓	F ✓
F	F	T	F ✓	T x	
F	F	F	F ✓	T x	

↑ possibilities for truth sayers/liars.

two possibilities

TFT and FTF