

Non-example $\begin{matrix} \in S^1 \\ \downarrow \\ \bigcirc S^1 \end{matrix} \quad \begin{matrix} \bigcirc \\ \downarrow \\ \bigcirc S^1 \end{matrix}$ is isomorphic even though both S^1 .

Remark $\otimes$ implies that f preserves the covering space structure, i.e. it takes $p_1^{-1}(x)$ to $p_2^{-1}(x)$ for each $x \in X$.

Example $F_2 = \pi_1(S^1 \vee S^1)$ $X = \begin{matrix} \xrightarrow{a} \\ \bigcirc \\ \xleftarrow{b} \end{matrix}$

cover  

subgp $\langle a \rangle$ $\langle b \rangle$


 $\langle bab^{-1} \rangle$

Construction of the universal cover $p: \tilde{X} \rightarrow X$ s.t. $p_*(\pi_1 \tilde{X})$ is trivial.

Examples

  

Thm X path connected, locally path connected, semi-locally simply connected, $\otimes$
then there is a simply connected cover $p: \tilde{X} \rightarrow X$.

Defn $\tilde{X} = \{ [r] \mid r \text{ is a path in } X \text{ starting at } x_0 \}$.

Remark define $p: \tilde{X} \rightarrow X$ by $[r] \mapsto r(1)$

X path connected $\Rightarrow p$ surjective.

need a topology on $\tilde{X}$: key point: let $\mathcal{U}$ be the collection of path connected open sets $U \subset X$ s.t. $\pi_1 U \rightarrow \pi_1 X$ is trivial. Then $\mathcal{U}$ is a basis for the topology on X if X is $\otimes$

Let $U[r] = \{ [r \cdot \gamma] \mid \gamma \text{ is a path in } U \text{ with } \gamma(0) = r(1) \}$

$p: U[r] \rightarrow U$

- surjective as U path connected
- injective as $\pi_1 U \rightarrow \pi_1 X$ trivial.

