MT1 Solutions

Q1 a) $(1,0), (-1,0)$ in truth set, $(0,0), (0,1)$ not

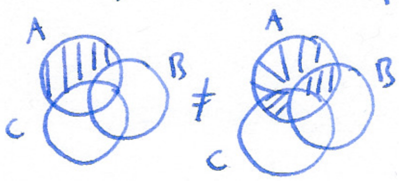
b) $\emptyset, \{0\}$ in truth set, $\mathbb{Z}, \{\ldots,-2,0,2,4,\ldots\}$ not.

Q2 c)

Q3 Thm The difference between the squares of consecutive integers is odd.

Proof Let $n, n+1$ be consecutive integers. Then $(n+1)^2 - n^2 = n^2+2n+1-n^2 = 2n+1$, which is odd. $\square$.

Q4 False, counterexample $A=\{1\}$, $B=\{1,2\}$, $C=\{1,3\}$.

Q5 

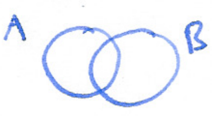
False, counterexample $A=\{1\}$, $B=\{1\}$, $C=\emptyset$.

Q6 Thm $B-A = A' \cap B$.

Proof $\subseteq$: Let $x \in B-A$, then $x \in B$ and $x \notin A$, so $x \in A'$, so $x \in A' \cap B$.

$\supseteq$: suppose $x \in A' \cap B$, then $x \in B$ and $x \in A'$, so $x \in B$, $x \notin A$, so $x \in B - A$. $\square$.

Q7 a) T b) VT c) T.

Q8 

$\underbrace{|A \cup B|}_{c} = \underbrace{|A|}_{a} + \underbrace{|B|}_{b} - |A \cap B|$ so $|A \cap B| = a+b-c$

so $|A-B| = a-(a+b-c)$
$= c-b$.

so $|P(A-B)| = 2^{c-b}$

Q9 Thm If $2 \mid a^2$ then $2 \mid a$.

Proof 2 cases: ① If a is even, then $a = 2n$ for some $n \in \mathbb{Z}$, then $a^2 = 4n^2$, even ✗

② if a is odd, then $a = 2n+1$, for $n \in \mathbb{Z}$, so $a^2 = 4n^2+4n+1$, odd ✓. $\square$.

Q10 Thm $P(A \cap B) = P(A) \cap P(B)$

Proof $\subseteq$: if $C \in P(A \cap B)$ then $C \subseteq A \cap B$, so $C \subseteq A$ and $C \subseteq B$, so $C \in P(A)$ and $C \in P(B)$, so $C \in P(A) \cap P(B)$.

$\supseteq$: if $C \in P(A)$ and $C \in P(B)$, then $C \subseteq A$ and $C \subseteq B$ so $C \subseteq A \cap B$, so $C \in P(A \cap B)$. $\square$.