Math 301 Introduction to Proof Fall 19 Sample Midterm 1

- (1) Find three distinct elements for the truth sets of the following statements:
 - (a) xy = 1, where the universe is $\mathbb{R} \times \mathbb{R}$.
 - (b) A is a subset of \mathbb{Z} which is closed under addition.
 - (c) A is an element of $\mathcal{P}(\mathbb{Z})$.
 - (d) A is a subset of $\mathcal{P}(\mathbb{Z})$.
 - (e) The numbers a, b and c are the lengths of the sides of a right angled triangle.
- (2) Consider the statement:

If a|c and b|c, then ab|c.

Which, if any, of the following substitutions give a counter example.

(a) a = 2, b = 3, c = 12(b) a = 3, b = 5, c = 24(c) a = 2, b = 2, c = 2

- (3) Write out a careful proof of the fact that the product of two odd numbers is odd.
- (4) What is $\mathcal{P}(\{\emptyset\})$? What is $\mathcal{P}(\mathcal{P}(\{\emptyset\}))$?
- (5) If $A \cup B \subseteq A \cup C$ does this imply that $B \subseteq C$?
- (6) State which of the following statements, are true, vacuously true, or false.
 (a) If B ⊆ A ∩ B, then B ⊆ A.
 (b) If P(A) = Ø, then A = Ø.
 - (c) If $A \in B$ and $B \in C$, then $A \in C$.
- (7) Suppose A and B are finite sets with |A| = a, |B| = B and |A ∪ B| = c. Find
 (a) |B \ A|
 (b) |A × (A ∪ B)|
 - (c) $|\mathcal{P}(A \cap B)|$

- (8) There is an island where all people either always lie, or always tell the truth. You meet three people, A, B and C.
 - ${\cal A}$ says: At least one of us is a truth teller.

B says: A is lying.

C says: B is lying.

What can you deduce?

(9) There is an island where all people either always lie, or always tell the truth. You meet D who says "I am lying." What can you deduce?