Math 505 Introduction to Proofs Spring 18 Sample Midterm 1

(1) Find three distinct elements of the truth set for the statement $3x - 4y = 4$, where the universe is $\mathbb{R} \times \mathbb{R}$.

(2) Consider the statement:

   If n is a square number, then n is not a cube.

   Which, if any, of the following substitutions give a counter example.
   (a) $n = 8$
   (b) $n = 64$
   (c) $n = 16$

(3) Write out a careful proof of the fact that the square of an odd number is odd.

(4) What is $\mathcal{P}(\varnothing)$? What is $\mathcal{P}(\mathcal{P}(\varnothing))$?

(5) If $A \cap B = A \cap C$ does this imply that $B = C$?

(6) State which of the following statements, are true, vacuously true, or false.
   (a) If $A \cap B \supseteq A$, then $A \subseteq B$.
   (b) If $\mathcal{P}(A) = \varnothing$, then $A = \varnothing$.
   (c) If $A \in B$ and $B \in C$, then $A \in C$.

(7) Suppose A and B are finite sets with $|A| = a$, $|B| = B$ and $|A \cap B| = c$. Find
   (a) $|B \setminus A|$
   (b) $|A \times (A \cap B)|$
   (c) $|\mathcal{P}(A \cap B)|$

(8) There is an island where all people either always lie, or always tell the truth. You meet three people, A, B and C.
   A says: At least two of us are truth tellers.
   B says: A is lying.
   C says: B is lying.
   What can you deduce?

(9) There is an island where all people either always lie, or always tell the truth. You meet D who says "I am lying." What can you deduce?