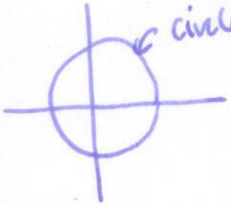
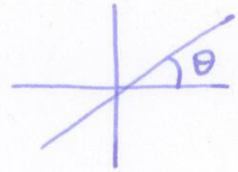
Equations in polar coordinates.



circle of radius R: cartesian $x^2+y^2=R^2$

polar $r=R$

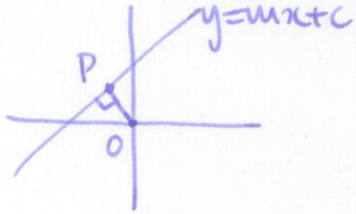


straight line through origin $y=mx$

$\theta=\tan^{-1}(m)$, $\theta=\tan^{-1}(m)+\pi$

convention: $(-r,\theta)=(r,\theta+\pi)$, so just need $\theta=\tan^{-1}(m)$.

general line:



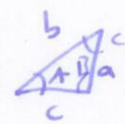
in polars: let P be closest point to origin $(0,0)$ and let $d(P,O)=d$.

$$\frac{d}{r}=\cos(\theta-\alpha) \qquad r=\frac{d}{\cos(\theta-\alpha)}=d\sec(\theta-\alpha)$$

cartesian: $(x-a)^2+y^2=a^2$

polar: $r=2a\cos\theta$

cosine rule: $a^2=b^2+c^2-2bc\cos\theta$



so $a^2=r^2+a^2-2ar\cos\theta \rightsquigarrow r^2=2ar\cos\theta \rightsquigarrow r=2a\cos\theta$

Examples sketch $r=\theta$

$r=\sin\theta$

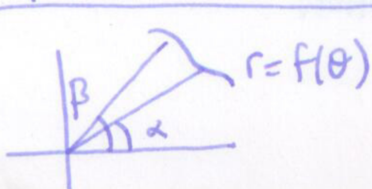
$r=\sin 2\theta$ etc.

can always try: $x^2+y^2=r^2$, ~~tan~~ $y/x=\tan\theta$ } $y=x^2 \rightsquigarrow r\sin\theta=r^2\cos^2\theta$ $\quad r=\frac{\sin\theta}{\cos^2\theta}=\tan\theta\sec\theta$.

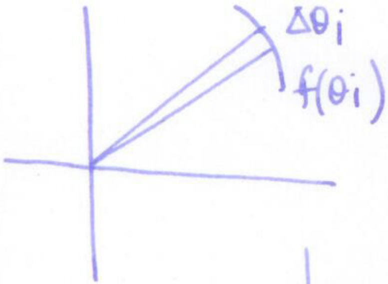
$x=r\cos\theta$

$y=r\sin\theta$

§11.4 Area and arc length in polars

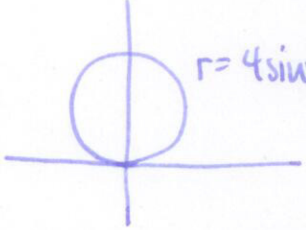


$$\text{area}=\int_\alpha^\beta \frac{1}{2}(f(\theta))^2\,d\theta$$



$$\text{area } \Delta A \approx \frac{\pi r^2}{2\pi/\Delta\theta_i} = \frac{1}{2} r^2 \Delta\theta_i$$

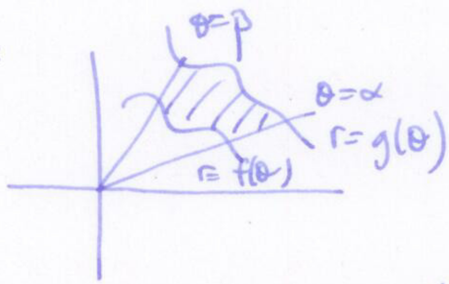
Example



$$\text{area} = \int_0^{\pi/2} \frac{1}{2}(4\sin\theta)^2 \, d\theta = \int_0^{\pi/2} 8\sin^2\theta \, d\theta$$

$$= 8\int_0^{\pi/2} \frac{1}{2}(1-\cos 2\theta)\,d\theta = \Big[4\theta - 2\sin 2\theta\Big]_0^{\pi/2} = 4\cdot\frac{\pi}{2} - 0 = 2\pi.$$

area between two curves:



$$\text{area} = \frac{1}{2}\int_\alpha^\beta (g(\theta))^2 - (f(\theta))^2 \, d\theta$$

arc length $\quad r = f(\theta)$ is parameterized curve with $\quad x = r\cos\theta = f(\theta)\cos\theta$
$\qquad y = r\sin\theta = f(\theta)\sin\theta$

so $$\frac{dx}{d\theta} = f'(\theta)\cos\theta + f(\theta)\cdot(-\sin\theta)$$

$$\frac{dy}{d\theta} = f'(\theta)\sin\theta + f(\theta)\cos\theta$$

arc length $$s = \int_\alpha^\beta \sqrt{(f'\cos\theta - f\sin\theta)^2 + (f'\sin\theta + f\cos\theta)^2}\, d\theta$$

$$\left.\begin{array}{l} (f')^2\cos^2\theta - 2f'f\cos\theta\sin\theta + f^2\sin^2\theta \\ (f')^2\sin^2\theta + 2ff'\sin\theta\cos\theta + f^2\cos^2\theta \end{array}\right\} = (f')^2 + f^2$$

so arc length $$s = \int_\alpha^\beta \sqrt{(f(\theta))^2 + (f'(\theta))^2}\, d\theta$$

Example $\quad$ circle $\quad r = 2a\cos\theta \quad f(\theta) = 2a\cos\theta$
$\qquad f'(\theta) = -2a\sin\theta$

$$\int_0^\pi \sqrt{4a^2\cos^2\theta + 4a^2\sin^2\theta}\, d\theta = \int_0^\pi 2a\, d\theta = \Big[2a\theta\Big]_0^\pi = 2\pi a.$$