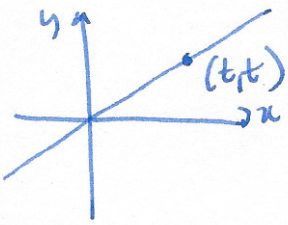
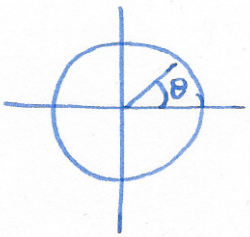
$= \int_0^\infty 2\pi\sqrt{1+u^2}\,du$ (trig sub integral)

§11.1 Parametric equations



$x(t)=t$
$y(t)=t$

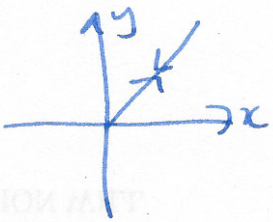
$c(t) = (x(t), y(t))$ e.g. ↑ parameterization



$c(\theta) = (\cos\theta, \sin\theta)$
$c(t) = (\cos t, \sin t)$

problem
~~Warning~~: many different parameterizations give the same curve

eg $c(t) = (2t, 2t)$ gives $y=x$, as does $c(t) = (t^3, t^3)$

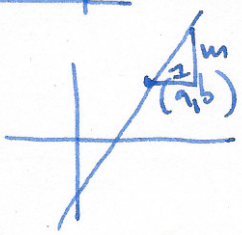


warning: $c(t) = (t^2, t^2)$ not the same!

Example: describe the parameterized curve $c(t) = (t+1, t^2+1)$ in terms of $y = f(x)$:

$x = t+1$
$y = t^2+1$
$\Rightarrow t = x-1$
$\Rightarrow y = (x-1)^2+1 = x^2-2x+2$

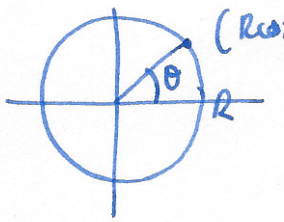
Example ① find a parametric equation for the line through (a,b) with slope m

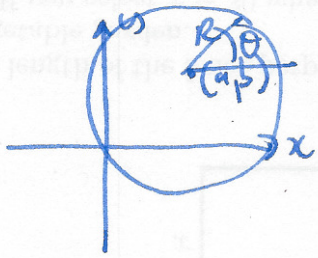


$t=0$: (a,b)
$t=1$: $(a+1, b+m)$

$c(t) = (a+t, b+tm)$

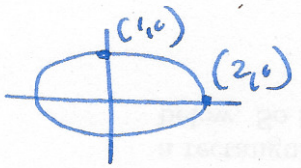
② circle of radius R, centered at (a,b)





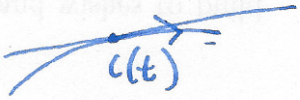
$c(\theta) = (a+R\cos\theta, b+R\sin\theta)$

③ ellipse



$c(\theta) = (2\cos\theta, \sin\theta)$

Q: how do we find the slope of the tangent line?



A: $\boxed{\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}}$