observation: $\int_a^b uv'dx = [uv]_a^b - \int_a^b u'v\,dx$

(2) $\int_1^2 \ln(x)dx = \int_1^2 \underbrace{1}_{v'} . \underbrace{\ln(x)}_{u} dx$ $\qquad u = \ln(x) \quad u' = \frac{1}{x}$

$\qquad v' = 1 \quad v = x$

$= \left[x\ln(x)\right]_1^2 - \int_1^2 x.\frac{1}{x}dx$

$= 2\ln(2) - 1\ln(1) - [x]_1^2 = 2\ln(2) - 1$

(3) $\int \underbrace{x^2}_{u} \underbrace{\cos(x)}_{v'} dx = x^2\sin(x) - \int \underbrace{2x}_{u'} \underbrace{\sin(x)}_{v} dx$

$= x^2\sin(x) - 2x(-\cos x) + \int 2(-\cos x)dx$

$= x^2 + 2x\cos(x) - 2\sin x + c$ $\qquad$ check!

(4) $\int \underbrace{e^x}_{u} \underbrace{\sin x}_{v'} dx = e^x(-\cos x) - \int \underbrace{e^x}_{u} \underbrace{(-\cos x)}_{v'} dx$

$\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$

$2\int e^x \sin x\,dx = e^x(\sin x - \cos x)$

$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + c$ $\qquad$ check!

§7.2 Trig integrals

$\int \sin^m x \cos^n x\,dx$?

tools:
- $\cos^2 x + \sin^2 x = 1$
- $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- sub $u = \sin x \quad \frac{du}{dx} = \cos x$.
- sub $u = \cos x \quad \frac{du}{dx} = -\sin x$
- parts.

Examples

- $\int \sin^2 x\, dx = \int \frac{1}{2} - \frac{1}{2}\cos 2x\, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$ (check!)

- $\int \sin^3 x\, dx = \int (1-\cos^2 x)\sin x\, dx$ sub $u = \cos x$, $\frac{du}{dx} = -\sin x$

$$= \int (1-u^2)\sin x \cdot \frac{1}{-\sin x}\, du = -\int 1-u^2\, du = -u + \frac{1}{3}u^3 + c$$

$$= -\cos x + \frac{1}{3}\cos^3 x + c \quad \text{(check!)}$$

moral: squares: double angle formula
odd powers: do sub $u =$ other trig function.

Example $\int \sin^4 x \cos^3 x\, dx$ try $u = \sin x$, $\frac{du}{dx} = \cos x$

$$\int \sin^4 x \cos^3 x\, dx = \int \sin^4 \cos^2 x \cdot \cos x\, dx = \int u^4(1-u^2)\, du = \int u^4 - u^6\, du = \frac{1}{5}u^5 - \frac{1}{7}u^7 +$$

$$= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c.$$

even powers $\int \sin^4 x \cos^2 x\, dx$ ← get everything in terms of $\sin(x)$ or $\cos(x)$, then use parts.

$$\int \sin^4 x (1-\sin^2 x)\, dx = \int \sin^4 x - \sin^6 x\, dx$$

$$\int \sin^6 x\, dx = \int \underbrace{\sin^5 x}_{u}\ \underbrace{\sin x}_{v'}\, dx$$

$u = \sin^5 x$ $\quad u' = 5\sin^4 x \cdot \cos x$

$v' = \sin x$ $\quad v = -\cos x$

$$= uv - \int u'v\, dx$$

$$= -\sin^5 x \cos x + \int 5\sin^4 x \cos^2 x\, dx$$

$$= -\sin^5 x \cos x + 5\int \sin^4 x (1-\sin^2 x)\, dx$$

$$\int \sin^6 x\,dx = -\sin^5 x \cos x + 5\int \sin^4 x\,dx - 5\int \sin^6 x\,dx$$

$$6\int \sin^6 x\,dx = -\sin^5 x \cos x + \underbrace{5\int \sin^4 x\,dx}_{\text{do by parts!}}$$

other trig functions

recall: $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$ $\quad u = \cos x$, $\frac{du}{dx} = -\sin x$ $\quad = \int \frac{\sin x}{u} \cdot \frac{-1}{\sin x}\,du$

$$= -\int \frac{1}{u}\,du = -\ln|u| + c = -\ln|\cos x| + c = \ln|\sec x| + c$$

fact: $\int \sec(x)\,dx = \ln|\sec x + \tan x| + c$ $\quad$ (check: $\frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$).

$$\int \operatorname{cosec} x\,dx = -\ln|\operatorname{cosec} + \cot x| + c$$

other trig function powers

$\int \tan^a x \sec^b x\,dx$ $\quad$ use

- $\cos^2 x + \sin^2 x = 1 \Longleftrightarrow 1 + \tan^2 x = \sec^2 x$
- $u = \sec x$ $\quad \frac{du}{dx} = \sec x \tan x$
- $u = \tan x$ $\quad \frac{du}{dx} = \sec^2 x$
- parts

a odd: $\int \tan^3 x \sec^2 x\,dx = \int \tan^2 x \sec x\,(\tan x \sec x)\,dx$

$= \int (1 - \sec^2 x) \sec x\,(\tan x \sec x)\,dx$ $\quad u = \sec x$, $\frac{du}{dx} = \sec x \tan x$

$= \int (1 - u^2) u\,du = \frac{1}{2}u^2 - \frac{1}{4}u^4 + c = \frac{1}{2}\sec^2 x - \frac{1}{4}\sec^4 x + c$

b even: $\int \tan^3 x \sec^2 x\,dx$ $\quad u = \tan x$, $\frac{du}{dx} = \sec^2 x$

$$\int u^3 \cdot \frac{\sec^2 x}{\sec^2 x}\,du = \frac{1}{4}u^4 + c = \frac{1}{4}\tan^4 x + c.$$