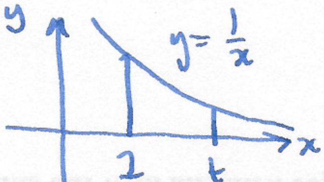
Example



between 1 and ∞

1 and t, let $t \to \infty$.

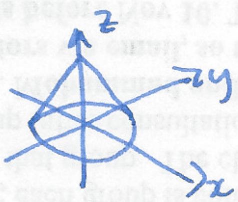
volume $V = \int_1^t \frac{\pi}{x^2}\,dx = \pi\left[-\frac{1}{x}\right]_1^t = \pi\left(1-\frac{1}{t}\right) \to \pi$ as $t \to \infty$

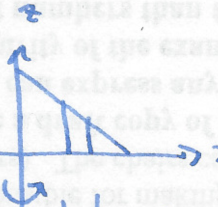
surface area $A = 2\pi\int_1^t \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx > 2\pi\int_1^t \frac{1}{x}\,dx = 2\pi\left[\ln|x|\right]_1^t$

$= 2\pi\ln(t) \to \infty$ as $t \to \infty$.

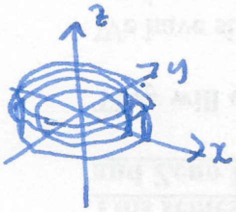
§6.4 Cylindrical shells

volume of cone:





rotate around y-axis.

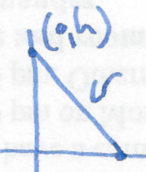


volume of shell $V_i \approx 2\pi x_i y_i \Delta x_i$

total volume $V \approx \sum V_i = \sum 2\pi x_i y_i \Delta x_i$

$$V = 2\pi\int_a^b x f(x)\,dx$$

Example



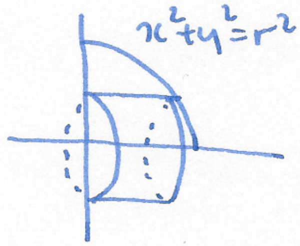
$xh + yr = hr$

$V = \int_0^r 2\pi x\left(\frac{hr - xh}{r}\right)dx$

$V = \frac{2\pi h}{r}\int_0^r x(r-x)\,dx$

$= \frac{2\pi h}{r}\int_0^r xr - x^2\,dx = \frac{2\pi h}{r}\left[\frac{1}{2}x^2 r - \frac{1}{3}x^3\right] = 2\pi\frac{h}{r}\left(\frac{1}{2}r^3 - \frac{1}{3}r^3\right) = \frac{1}{3}\pi r^2.$

Example



volume of hemisphere : rotate horizontal rectangle about x-axis.

$$V = \int 2\pi y\, f(y)\, dy$$

$$V = \int_0^r 2\pi y \sqrt{r^2-y^2}\, dy = 2\pi \left[\frac{2}{3}(r^2-y^2)^{3/2} \cdot -\frac{1}{2} \right]_0^r = -\frac{2\pi}{3}(0-r^3) = \frac{2}{3}\pi r^3.$$

§7.1 Integration by parts

"reverse product rule" product rule : $(uv)' = u'v + uv'$

$$\int (uv)'\,dx = \int u'v + uv'\,dx$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\boxed{\int uv'\,dx = uv - \int u'v\,dx}$$ ← integration by parts formula

Example $\int x\sin(x)\,dx$ $u = x$ $v' = \sin(x)$
$u' = 1$ $v = -\cos(x)$

$$\int \underbrace{x}_{u}\underbrace{\sin(x)}_{v'}\,dx = \underbrace{x}_{u}\underbrace{(-\cos x)}_{v} - \int \underbrace{(1)}_{u'}\underbrace{(-\cos x)}_{v}\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$ check!

Examples ① $\int \underbrace{x}_{u}\underbrace{e^x}_{v'}\,dx$ $u = x$ $v' = e^x$
$u' = 1$ $v = e^x$ } Q: spose we chose these the other way round?

$$= uv - \int u'v\,dx = xe^x - \int 1e^x\,dx = xe^x - e^x + C.$$ check!