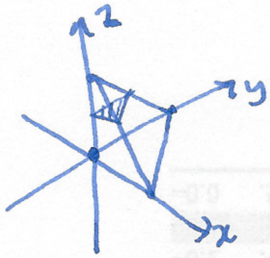
Example: tetrahedron with vertices $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(1,0,0)$



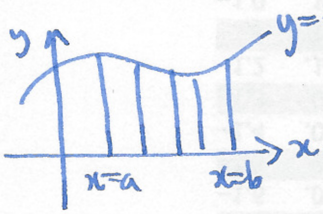
$$V = \int_0^1 A(z)\,dz \qquad \text{area of triangle} = \tfrac{1}{2}\,\text{base} \times \text{height}$$

$$= \tfrac{1}{2}(1-z)(1-z)$$

$$= \int_0^1 \tfrac{1}{2}(1-z)^2\,dz = \cdots = \tfrac{1}{6}$$

recall: average value of numbers $a_1, \dots, a_n = \dfrac{a_1 + \cdots + a_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} a_i$

Q: what about average value of function $f(x)$ on an interval $[a,b]$?



average: $\frac{1}{n}\left(f(x_1) + \dots + f(x_n)\right)$

recall: $\displaystyle\int_a^b f(x)\,dx \approx \left(f(x_1) + \dots + f(x_n)\right)\Delta x$, $\Delta x = \frac{b-a}{n}$

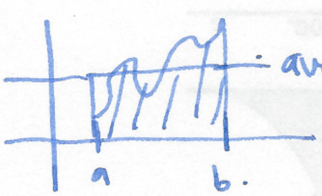
so $\displaystyle\int_a^b f(x)\,dx \approx \text{average } f(x)\,(b-a)$, so $\text{average} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Example find average value of $\sin(x)$ on $[0,\pi]$

$[0,\pi]$: $\displaystyle\frac{1}{\pi}\int_0^\pi \sin(x)\,dx = \frac{1}{\pi}\Big[-\cos(x)\Big]_0^\pi = \frac{1}{\pi}\left(-(-1)+1\right) = \frac{2}{\pi}$

useful fact (mean value theorem for integrals)

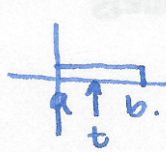
If $f(x)$ is cts on $[a,b]$, then there is a $c \in [a,b]$ s.t. $f(c) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$



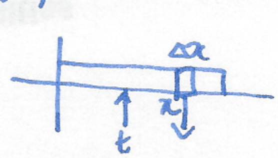
Example find average value of $\dfrac{\sin(\frac{\pi}{x})}{x^2}$ on $[1,2]$

Q: which has a bigger average on $[0,\pi]$, $\sin(x)$ or $\sin^2 x$?

Application: center of mass.



density $\rho(x)$



turning force $(x-t)\Delta x\,\rho(x)$.

turning force at t: $\displaystyle\sum_i \rho(x)(x_i - t)\Delta x_i \approx \int_a^b \rho(x)(x-t)\,dt = \int_a^b x\rho(x)\,dx - t\int_a^b \rho(x)\,dx = 0$ $\qquad t = \dfrac{\int_a^b x\rho(x)\,dx}{\int_a^b \rho(x)\,dx}$.