useful rules

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$

$f(x)$	$\int f(x)\,dx$	
x^n	$\frac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert$	
$\sin(x)$	$-\cos(x)$	
$\cos(x)$	$\sin(x)$	
e^x	e^x	

general rules

f	f'
$kf(x)$	$kf'(x)$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))\,g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x)f'(x)-g'(x)f(x)}{g(x)^2}$

f	$\int f(x)\,dx$
$kf(x)$	$k\int f(x)\,dx$
$f(x)+g(x)$	$\int f(x)\,dx + \int g(x)\,dx$

§5.7 Integration by substitution

$$\int_{x=a}^{x=b} f(x)\,dx \ , \ x(u) \quad \rightsquigarrow \quad \int_{u=x^{-1}(a)}^{u=x^{-1}(b)} f(x(u))\,\frac{dx}{du}\,du$$

recall $\frac{dx}{du} = \frac{1}{\frac{du}{dx}}$ integration by substitution is "reverse chain rule".

Examples $\bullet \int x\sin(x^2)\,dx$ try $x^2 = u$ $\frac{du}{dx} = 2x$

$$\int x\sin(x^2)\frac{dx}{du}\,du = \int x\sin(u)\frac{1}{2x}\,du = \int \frac{1}{2}\sin(u)\,du$$

$$= -\frac{1}{2}\cos(u) + c = -\cos(x^2)+c \qquad \text{check!}$$

- $\int e^{4x}\,dx$ $\quad u = 4x$, $\frac{du}{dx} = 4$ $\quad \int e^u \frac{1}{4}\,du = \frac{1}{4}e^u + c = \frac{1}{4}e^{4x} + c$ check!

- $\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\,d\theta$ $\quad \left[\text{hint: } \frac{d}{dx}\left(\ln(f(x))\right) = \frac{1}{f(x)}\cdot f'(x)\right]$

try: $u = \cos\theta$, $\frac{du}{d\theta} = -\sin\theta$

$$\int \frac{\sin\theta}{u}\cdot -\frac{1}{\sin\theta}\,d\theta = -\int \frac{1}{u}\,du = -\ln|u| + c$$
$$= -\ln|\cos x| + c$$
$$= \ln|\sec x| + c$$

check!

- $\int x\sqrt{x+1}\,dx$ $\quad$ try $u = x+1$, $\frac{du}{dx} = 1$ $\quad \int x\sqrt{u}\,du = \int (u-1)\sqrt{u}\,du$

$$= \int u^{3/2} - u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{2u^{3/2}}{3} + c = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$$

Definite integrals

- $\int_0^2 x^2\sqrt{x^3+1}\,dx$ $\quad u = x^3+1$, $\frac{du}{dx} = 3x^2$ $\quad \int_1^9 x^2\sqrt{u}\,\frac{1}{3x^2}\,du = \int_1^9 \frac{1}{3}u^{1/2}\,du = \left[\frac{2}{9}u^{3/2}\right]_1^9$

$$= \frac{2}{9}27 - \frac{2}{9}.$$

§5.8 More functions

inverse trig functions

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + c$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2} \qquad \int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + c$$

$$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + c$$

Example $\int_0^1 \frac{1}{4+x^2}\,dx$ $\quad$ try $x = 2u$, $x^2 = 4u^2$ $\quad \frac{dx}{du} = 2 \rightsquigarrow \int_0^{1/2} \frac{1}{4+4u^2}\,2\,du$

$$= \frac{1}{2}\int_0^{1/2} \frac{1}{1+u^2}\,du = \left[\frac{1}{2}\tan^{-1}(u)\right]_0^{1/2} = \frac{1}{2}\left(\tan^{-1}(1/2) - \tan^{-1}(0)\right) = \frac{1}{2}\tan^{-1}(1/2).$$

exponential functions

$$\frac{d}{dx}(e^x) = e^x \qquad \int e^x \, dx = e^x + c$$

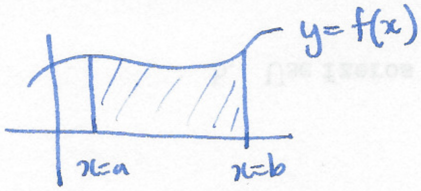
$$\frac{d}{dx}(b^x) = \frac{d}{dx}\left(e^{x\ln(b)}\right) = \ln(b)\, e^{x\ln(b)} = \ln(b)\, b^x$$

$$\int b^x \, dx = \frac{1}{\ln(b)} b^x + c$$

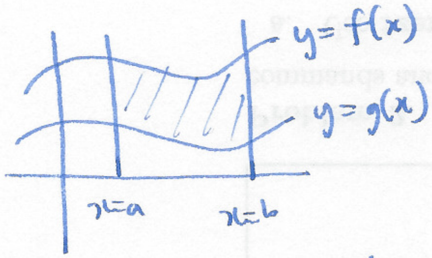
Examples $\int_0^1 10^x \, dx \qquad \int_0^{\pi/2} \cos\theta \, 10^{\sin\theta} \, d\theta.$

§6.1 Area between two curves

recall



area under graph $\int_a^b f(x)\,dx$



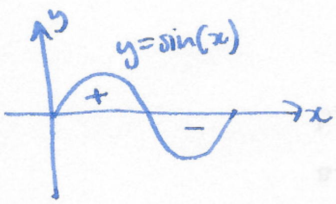
area between two curves

$$\int_a^b f(x) - g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

warning : signs !

$$\int_a^b f(x) - g(x)\,dx$$

Q: what do you do if you want "absolute area" / unsigned area.



$\int_0^{2\pi} \sin(x)\,dx = 0$ want: $\int_0^{2\pi} |\sin(x)|\,dx$ ← need to know where sin(x) +ve or −ve.

i.e. need to find zeros of $\sin(x) = 0$ $\int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x\,dx$

$$= \left[-\cos(x)\right]_0^{\pi} - \left[\cos(x)\right]_{\pi}^{2\pi}$$

$$= 2 - -2 = 4.$$