MTH 232 Calculus 2

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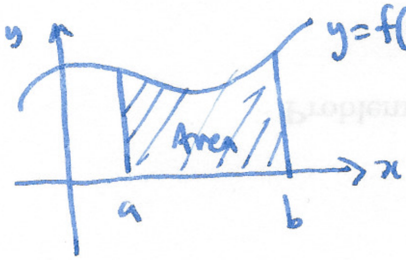
office 1S-222 office hours M 2:30 – 4:20
W 2:30 – 3:20

- math tutoring: 1S-214
- students w/ disabilities.

Text: Calculus, early transcendentals, Rogawski + Adams

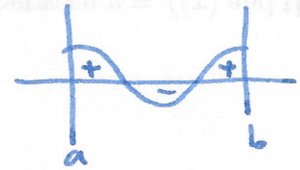
HW: webworks / Matlab projects / quizes

§5.2 Definite integral



intuition: $\int_a^b f(x)\,dx$ = area under the curve $y=f(x)$ between $x=a$ and $x=b$

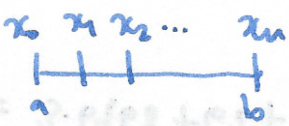
note: signed area



formal defn: Riemann sum $R(f,P,C) = \sum f(c_i)\Delta x_i$, $\Delta x_i = |x_{i} - x_{i-1}|$

f function

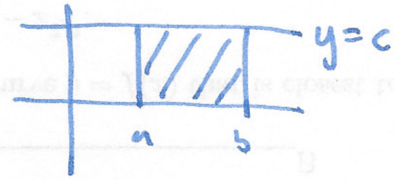
P partition of $[a,b]$



c choice of point in $c_i \in [x_{i-1}, x_i]$

$$\int_a^b f(x)\,dx = \lim_{\|P\| \to 0} R(f,P,C) \qquad \|P\| = \max \Delta x_i$$

useful properties $\int_a^b c\,dx = c(b-a)$



$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

reversing limits: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

0-length intervals: $\int_a^a f(x)\,dx = 0$

adjacent intervals: $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

§5.3 Indefinite integrals

Defn A function $F(x)$ is an anti-derivative for $f(x)$ if $F'(x) = f(x)$.

General antiderivative: if $F(x)$ is an anti-derivative for $f(x)$, then any other antiderivative is of the form $F(x)+C$ for some constant c.

⌈Proof: spose f has antiderivatives F, H. Then $(F-H)' = F'-H' = f-f = 0$

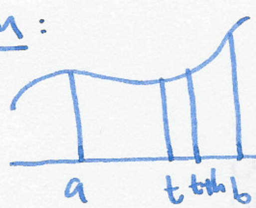
$\Rightarrow$ $F-H =$ constant function⌋

notation: $\int f(x)\,dx = F(x)+c$ means $F(x)+c$ general antiderivative for $f(x)$

§5.4 Fundamental theorem of calculus I

Thm (FTC ①) suppose $f(x)$ is cts on $[a,b]$ and $F(x)$ is an antiderivative for $f(x)$, i.e. $F'(x) = f(x)$. Then $\int_a^b f(x)\,dx = F(b)-F(a)$

intuition:



consider $\int_a^t f(x)\,dx$ ← function of t!

Q: what is the rate of change wrt t?

recall $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ so $\frac{d}{dt}\left(\int_a^t f(x)\,dx\right)$

$$= \lim_{h\to 0} \frac{\int_a^{t+h} f(x)\,dx - \int_a^t f(x)\,dx}{h} = \lim_{h\to 0} \frac{\int_t^{t+h} f(x)\,dx}{h} \approx \frac{f(t)\times h}{h}$$

area of rectangle

$\approx f(t)$

i.e. $\int_a^t f(x)\,dx$ is an antiderivative for $f(x)$, so $\int_a^t f(x)\,dx = F(t)+c$

Q: what is the constant? $t=a$: $\int_a^a f(x) = 0 = F(a)+c \Rightarrow c=-F(a)$

so $\int_a^t f(x)\,dx = F(t) - F(a)$ $\square$

<u>Example</u> $\int_2^3 \sqrt{x} + \frac{1}{x} + \sin(x)\,dx = \left[\frac{2x^{3/2}}{3} + \ln|x| - \cos(x)\right]_2^3$

$= \frac{3(3)^{2/3}}{3} + \ln|3| - \cos(3) - \left(\frac{2\cdot 2^{3/2}}{3} + \ln|2| - \cos(2)\right)$

<u>§5.5 Fundamental theorem of calculus II</u>

<u>Thm</u> (FTC②) let $f(x)$ be a cts function on $[a,b]$, then $A(x) = \int_a^x f(t)\,dt$ is an antiderivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx} \Leftrightarrow \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Furthermore: $A(a) = 0$

<u>Example</u> $\int_0^x e^{-t^2}\,dt \leftarrow$ a function with derivative e^{-x^2}.

<u>Example</u> what about $\int_0^{x^2} \sin(t)\,dt \leftarrow$ function of a function

to find $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$ set $A(x) = \int_0^x \sin(t)\,dt$, then $A'(x) = \sin(t)$

so $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt = \frac{d}{dx}\left(A(x^2)\right) \underset{\text{chain rule}}{=} A'(x^2)\cdot(x^2)' = A'(x^2)\cdot 2x = \sin(x^2)\cdot 2x$

<u>Aside</u>: when people say "not every formula can be integrated" what do they mean? If $f(x)$ cts, then $A(x) = \int_0^x f(t)\,dt$ is an integral for $f(x)$, but we might not be able to write it as a formula involving basic functions.

Analogy
- $\sqrt{2} \approx 1.414\ldots$ is a real number, but not a fraction
- $x^5 - x - 1$ has 1 real root which cannot be written as an expression involving rational numbers and fractional powers [Galois theory]
- e^{-x^2} has an integral that can't be written as a combination of elementary functions [differential Galois theory]