Math 232 Calculus 2 Spring 18 Midterm 2b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int \sin 4x \sin 3x \, dx$.

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$

$= \frac{1}{2} \int \overset{\cos}{\cancel{\sin}} x - \overset{\cos}{\cancel{\sin}} 7x \, dx = \frac{1}{2} \sin x - \frac{1}{14} \sin 7x + C$

(2) (10 points) Find $\int \sec^3 x\, dx.$ $= \int \sec x \cdot \sec^2 x\, dx$

$\int uv'\,dx = uv - \int u'v\,dx$ $\quad u = \sec x \quad u' = \sec x \tan x \quad \sin^2 x + \cos^2 x = 1$

$v' = \sec^2 x \quad v = \tan x \quad \tan^2 x + 1 = \sec^2 x$

$= \sec x \tan x - \int \sec x \underbrace{\tan^2 x}_{\sec^2 x - 1}\, dx$

$\int \sec^3 x\, dx = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

$2\int \sec^3 x\, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$

$\int \sec^3 x\, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

$\sin^2 x + \cos^3 x = 1$

(3) (10 points) Find $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx.$

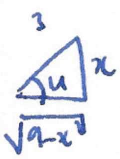
$x = 3\sin u$

$\dfrac{dx}{du} = 3\cos u$

$$= \int \frac{9\sin^2 u}{\sqrt{9-9\sin^2 u}} \cdot \frac{dx}{du}\,du = \int \frac{9\sin^2 u}{3\cos u}\, 3\cos u\, du = \int 9\sin^2 u\, du$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$$= \frac{9}{2}\int 1 - \cos 2u\, du = \frac{9}{2}u - \frac{9}{4}\sin 2u + c = \frac{9}{2}u - \frac{9}{2}\sin u\cos u + c$$



$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{2}\cdot\frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{3} + c$$

(4) (10 points) Find $\int \frac{x^2+x+2}{(x+1)(x^2+1)}\,dx.$

$$\frac{x^2+x+2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1) + \cancel{B(x+1)}\ (Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$x=-1$: $2 = 2A \Rightarrow A=1$

$x=0$: $2 = A + C \Rightarrow C = 1$

$x=1$: $4 = 2A + 2B + 2C \Rightarrow B=0$

$$\int \frac{1}{x+1} + \frac{1}{x^2+1}\,dx = \ln|x+1| + \tan^{-1}(x) + c$$

$\int uv'dx = uv - \int u'v dx$

(5) (10 points) Find $\int_0^\infty xe^{-2x}\,dx$.

$\int \underset{u}{x}\,\underset{v'}{e^{-2x}}\,dx$ $\quad u = x \quad u' = 1$

$v' = e^{-2x} \quad v = -\frac{1}{2}e^{-2x}$

$= -\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}\,dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$

$\lim_{R\to\infty} \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} \right]_0^R = \lim_{R\to\infty} -\frac{1}{2}Re^{-2R} - \frac{1}{4}e^{-2R} + 0 + \frac{1}{4} = \frac{1}{4}$

(6) (10 points) Find the degree three Taylor polynomial for $f(x) = e^{1/x}$ centered at $x = 1$.

$f(x) = e^{x^{-1}}$ $\qquad f(1) = e$

$f'(x) = e^{x^{-1}} \cdot -x^{-2}$ $\qquad f'(1) = -e$

$f''(x) = e^{x^{-1}} \cdot x^{-4} + e^{x^{-1}} \cdot 2x^{-3}$ $\qquad f''(1) = 3e$

$f^{(3)}(x) = e^{x^{-1}} \cdot -x^{-6} + e^{x^{-1}} \cdot -4x^{-5} + e^{x^{-1}} \cdot -2x^{-5} + e^{x^{-1}} \cdot -6x^{-4}$ $\qquad f^{(3)}(1) = -13e$

$T_3(x) = e - e(x-1) + 3e\frac{(x-1)^2}{2!} + -13e\frac{(x-1)^3}{3!}$

(7) Consider the series $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} \cdots$. Find an explicit formula for the partial sum S_N. Hint: compare S_N with $\frac{2}{3}S_N$. Use your formula for S_N to show that the series converges, and find the limit.

$$S_N = 1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots + \left(\frac{2}{3}\right)^{N-1}$$

$$\frac{2}{3}S_N = \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots + \left(\frac{2}{3}\right)^{N-1} + \left(\frac{2}{3}\right)^N$$

$$S_N - \frac{2}{3}S_N = 1 - \left(\frac{2}{3}\right)^N$$

$$\frac{1}{3}S_N = 1 - \left(\frac{2}{3}\right)^N \qquad S_N = 3\left(1 - \left(\frac{2}{3}\right)^N\right)$$

$$\lim_{N\to\infty} 3\left(1 - \left(\frac{2}{3}\right)^N\right) = 3.$$

(8) Does the series $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$ converge or diverge? If it converges, find the exact value.

$$\frac{1}{n^2+3n+2} = \frac{1}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2} = \frac{A(n+2)+B(n+1)}{(n+1)(n+2)}$$

$n=-1$: $1 = A$

$n=-2$: $1 = -B$

$$= \frac{1}{n+1} - \frac{1}{n+2}$$

$$S_N = \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{N+1} - \frac{1}{N+2} = \frac{1}{2} - \frac{1}{N+2}$$

$$\lim_{N\to\infty} \frac{1}{2} - \frac{1}{N+2} = \frac{1}{2}$$

(9) Does the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ converge or diverge? Explain.

$$1 + \underbrace{\frac{1}{2}}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{\geq \frac{1}{2}} + \underbrace{\frac{1}{9} + \cdots + \frac{1}{16}}_{\geq \frac{1}{2}} + \cdots$$

diverges.

(10) Does the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ converge or diverge?

$\lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$ so diverges.