Math 232 Calculus 2 Spring 18 Sample midterm 2

(1) Find $\int \tan^3 2x \; dx$.

(2) Find $\int \cos 11x \sin 7x \; dx$.

(3) Find $\int \dfrac{x}{\sqrt{4x^2+1}} dx$.

(4) Find $\int \dfrac{5x+4}{(x-2)(x+2)^2} dx$.

(5) Find $\int_0^1 x^2 \ln x^2 \; dx$.

(6) Find $\int_0^\infty \dfrac{1}{16+x^2} \; dx$.

(7) Can you find the degree three Taylor polynomial centered at $x = 0$ for the function $f(x) = \sqrt{x}$, why or why not? Find the degree three Taylor polynomial for this function centered at $x = 1$. Find an error bound for the approximation for $\sqrt{2}$.

(8) Does the sequence $a_n = \dfrac{3^n}{n!}$ converge or diverge?

(9) Does the series $\sum_{n=2}^{\infty} e^{-n}$ converge or diverge? If it converges, find the exact value.

(10) Does the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2+3n+2}$ converge or diverge? If it converges, find the exact value.

(11) Does the series $\sum_{n=1}^{\infty} \cos(\frac{1}{n})$ converge or diverge?

(12) Bonus question: suppose you want to approximate $\sqrt{2}$ using Newton's method. Start with $f(x) = x^2 - 2$ and $x_1 = 2$. Show that

$$x_{n+1} = \frac{x_n^2 + 2}{2x_n}.$$

Show that this sequence is decreasing and bounded below by $\sqrt{2}$. Can you show that it converges to $\sqrt{2}$?