Math 232 Calculus 2 Spring 18 Midterm 1b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int xe^{x^2+1}\,dx$.

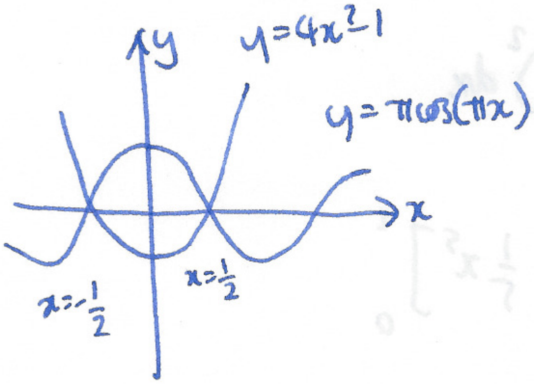
$u = x^2+1$

$\frac{du}{dx} = 2x$

$$\int x e^u \frac{dx}{du}\,du = \int xe^u \cdot \frac{1}{2x}\,du = \frac{1}{2}\int e^u\,du$$

$$= \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2+1} + c$$

(2) (10 points) Find the area bounded between the curves $y = \pi\cos(\pi x)$ and $y = 4x^2 - 1$.



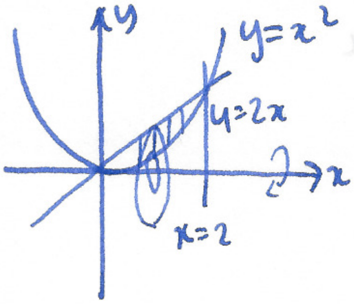
$$\int_{-1/2}^{1/2} \pi\cos(\pi x) - (4x^2-1)\,dx$$

$$= \left[\sin(\pi x) - \frac{4}{3}x^3 + x \right]_{-1/2}^{1/2}$$

$$= 1 - \frac{4}{3}\cdot\frac{1}{8} + \frac{1}{2} - \left(-1 - \frac{4}{3}\cdot\frac{1}{-8} - \frac{1}{2}\right)$$

$$= 1 - \frac{1}{6} + \frac{1}{2} + 1 - \frac{1}{6} + \frac{1}{2} = 3 - \frac{1}{3} = \frac{8}{3}$$

(3) (10 points) Draw a picture of the region bounded by the curves $y = x^2$ and $y = 2x$. Find the volume of revolution of this region about the x-axis.



discs

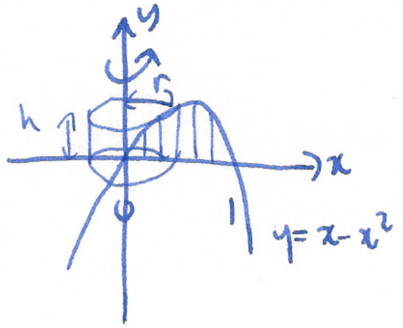
$$\int_0^2 \pi r^2\,dx = \pi \int_0^2 (\cancel{x^2})^2 - (x^2)^2\,dx$$

$2x$

$$= \pi \int_0^2 4x^2 - x^4\,dx = \pi\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^2$$

$$= \pi\left(\frac{32}{3} - \frac{32}{5}\right) = 32\pi \cdot \frac{2}{15} \cdot = \frac{64\pi}{15} \approx 13.40$$

(4) (10 points) Use shells to find the volume of revolution of the region given by $y = x - x^2$ with $y \geq 0$, rotated about the y-axis.

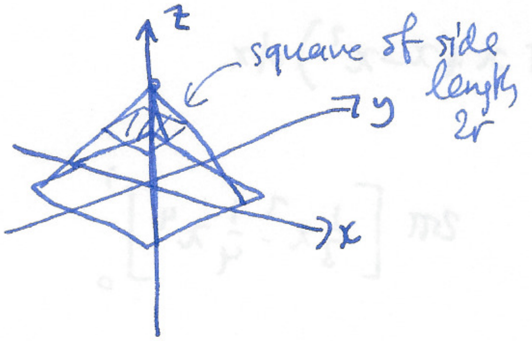


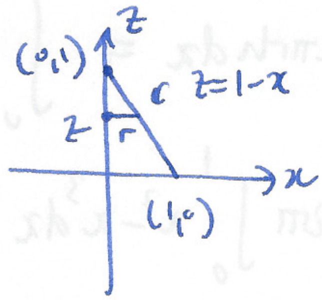
$$\int_0^1 2\pi r h \, dx = \int_0^1 2\pi x . (x - x^2) dx$$

$$= 2\pi \int_0^1 x^2 - x^3 dx = 2\pi \left[\frac{1}{3}x^3 - \frac{1}{4}x^4 \right]_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{4} \right)$$

$$= \frac{\pi}{6}$$

(5) (10 points) Consider the pyramid whose base is the square of side length 2 in the xy-plane, centered at the origin, and whose top vertex is $(0,0,1)$. Find a formula for the horizontal cross-sectional area at height z, and then use this to find the volume of the pyramid.





$$A = (2r)^2 = 4r^2 = 4x^2$$
$$= 4(1-z)^2$$

$$V = \int_0^1 A(z)\,dz = \int_0^1 4(1-z)^2\,dz = 4\int_0^1 1-2z+z^2\,dz$$

$$= 4\left[z - z^2 + \frac{1}{3}z^3\right]_0^1 = 4\left(1-1+\frac{1}{3}\right) = \frac{4}{3}$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(6) (10 points) Find $\int e^{-x}\cos(2x)\,dx$.

$$\int \underset{u}{e^{-x}}\,\underset{v'}{\cos(2x)}\,dx \qquad u = e^{-x} \quad u' = -e^{-x}$$

$$v' = \cos(2x) \quad v = \tfrac{1}{2}\sin(2x)$$

$$= \int e^{-x}\tfrac{1}{2}\sin(2x) - \int -e^{-x}\cdot\tfrac{1}{2}\sin 2x\,dx$$

$$= \tfrac{1}{2}e^{-x}\sin(2x) + \tfrac{1}{2}\int \underset{u}{e^{-x}}\,\underset{v'}{\sin 2x}\,dx \qquad u = e^{-x} \quad u' = -e^{-x}$$

$$v' = \sin 2x \quad v = -\tfrac{1}{2}\cos 2x$$

$$\int e^{-x}\cos(2x)\,dx = \tfrac{1}{2}e^{-x}\sin(2x) + \tfrac{1}{2}e^{-x}\cdot -\tfrac{1}{2}\cos(2x) - \tfrac{1}{2}\int -e^{-x}\cdot -\tfrac{1}{2}\cos 2x\,dx$$

$$\tfrac{5}{4}\int e^{-x}\cos(2x)\,dx = \tfrac{1}{2}e^{-x}\sin(2x) - \tfrac{1}{4}e^{-x}\cos(2x) + c$$

$$\int e^{-x}\cos(2x)\,dx = \tfrac{2}{5}e^{-x}\sin(2x) - \tfrac{1}{5}e^{-x}\cos(2x) + c$$

(7) Find $\displaystyle\int_0^{\pi/2} \sin^3 x \cos x \, dx$.

$u = \sin^3 x$

$\dfrac{du}{dx} = \cos x$

$$\int_0^1 u^3 \cos x \cdot \frac{dx}{du} \, du = \int_0^1 u^3 \cdot \cos x \, \frac{1}{\cos x} \, du$$

$$= \int_0^1 u^3 \, du = \left[\frac{1}{4}u^4\right]_0^1 = \frac{1}{4}$$

(8) Find $\int \sin(7x)\sin(5x)\,dx$.

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\cos(A+B) \mp \cos(A-B) = -2\sin A\sin B$$

$$= \frac{1}{2}\int \cos(2x) - \cos(12x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{24}\sin(12x) + C$$

(9) Find $\int \frac{1}{1+4x^2}\,dx.$ $= \int \frac{1}{1+(2x)^2}\,dx$

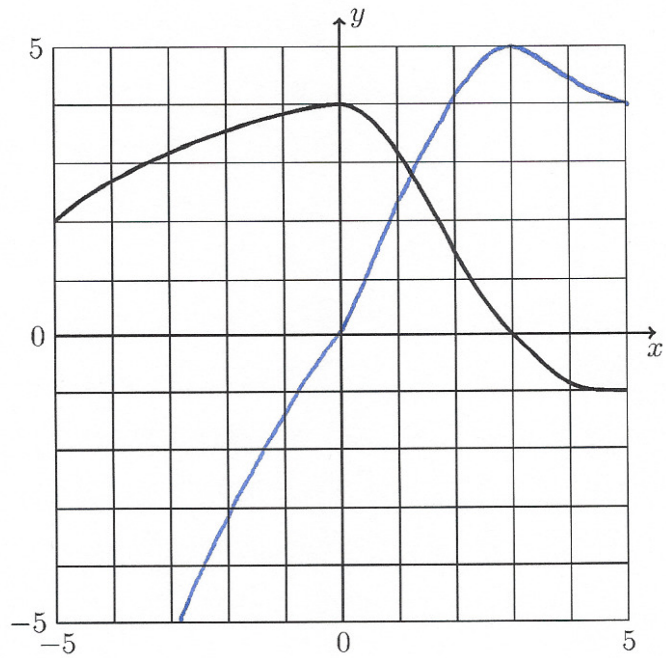
$u = 2x$

$\frac{du}{dx} = 2$

$\int \frac{1}{1+u^2} \cdot \frac{dx}{du}\,du = \int \frac{1}{1+u^2} \cdot \frac{1}{2}\,du$

$= \frac{1}{2}\tan^{-1}(u) + c = \frac{1}{2}\tan^{-1}(2x) + c$

(10) Consider the graph of the function f drawn below.



Let $g(x) = \int_0^x f(t)\, dt$.
Sketch $g(x)$, and find $g(0)$, $g'(0)$ and $g'(3)$.

$g(0) = 0$
$g'(0) = 4$
$g'(3) = 0$