Math 232 Calculus 2 Spring 18 Midterm 1a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int xe^{x^2-1}\,dx$.

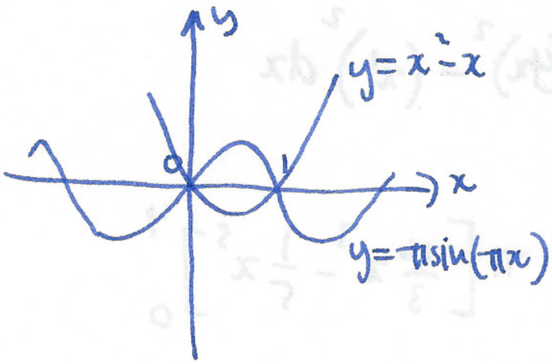
$u = x^2 - 1$

$\frac{du}{dx} = 2x$

$\int x e^u \frac{dx}{du}\,du = \int x e^u \cdot \frac{1}{2x}\,du$

$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2-1} + c$

(2) (10 points) Find the area bounded between the curves $y = \pi \sin(\pi x)$ and $y = x^2 - x$. $= x(x-1)$

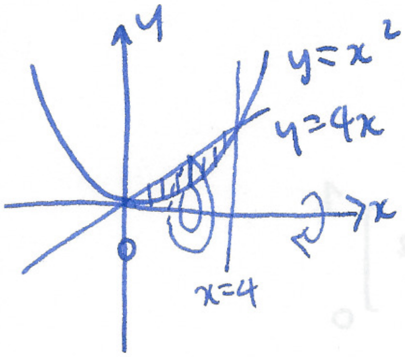


$$\int_0^1 \pi \sin(\pi x) - x^2 + x \, dx$$

$$\left[-\cos(\pi x) - \frac{1}{3}x^3 + \frac{1}{2}x^2 \right]_0^1$$

$$= -(-1) - \frac{1}{3} + \frac{1}{2} - (-1) = 2 + \frac{1}{6} = \frac{13}{12}$$

(3) (10 points) Draw a picture of the region bounded by the curves $y = x^2$ and $y = 4x$. Find the volume of revolution of this region about the x-axis.



disc:

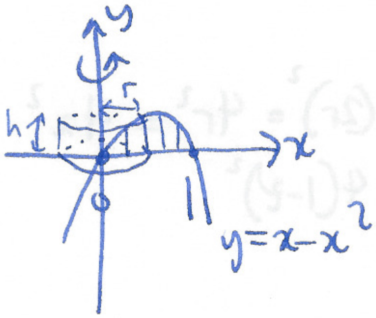
$$\int_0^4 \pi r^2 \, dx = \pi \int_0^4 (4x)^2 - (x^2)^2 \, dx$$

$$= \pi \int_0^4 16x^2 - x^4 \, dx = \pi \left[\frac{16}{3}x^3 - \frac{1}{5}x^5 \right]_0^4$$

$$= \pi\left(\left(\frac{16 \cdot 64}{3} - \frac{4^5}{5} \right) - (0) \right) = 4^5\pi \left(\frac{1}{3} - \frac{1}{5} \right) = 4^5\pi \cdot \frac{2}{15}$$

$$= \frac{2048\pi}{15} \approx 428.93$$

(4) (10 points) Use shells to find the volume of revolution of the region given by $y = x - x^2$ with $y \geq 0$, rotated about the y-axis.

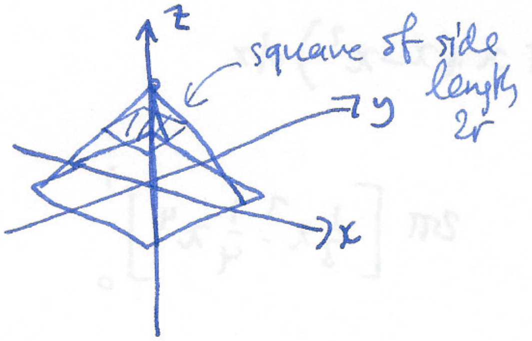


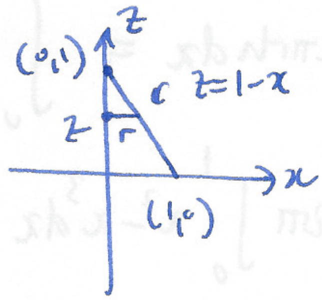
$$\int_0^1 2\pi r h \, dx = \int_0^1 2\pi x (x - x^2) \, dx$$

$$= 2\pi \int_0^1 x^2 - x^3 \, dx = 2\pi \left[\frac{1}{3}x^3 - \frac{1}{4}x^4 \right]_0^1$$

$$= 2\pi \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{\pi}{6}$$

(5) (10 points) Consider the pyramid whose base is the square of side length 2 in the xy-plane, centered at the origin, and whose top vertex is $(0, 0, 1)$. Find a formula for the horizontal cross-sectional area at height z, and then use this to find the volume of the pyramid.





$$A = (2r)^2 = 4r^2 = 4x^2 = 4(1-z)^2$$

$$V = \int_0^1 A(z)\,dz = \int_0^1 4(1-z)^2\,dz = 4\int_0^1 1-2z+z^2\,dz$$

$$= 4\left[z - z^2 + \frac{1}{3}z^3\right]_0^1 = 4\left(1 - 1 + \frac{1}{3}\right) = \frac{4}{3}$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(6) (10 points) Find $\int e^{-x}\sin(3x)\,dx$.

$$\int \underset{u}{e^{-x}}\,\underset{v'}{\sin(3x)}\,dx$$

$u = e^{-x}$ $u' = -e^{-x}$

$v' = \sin(3x)$ $v = -\frac{1}{3}\cos(3x)$

$$= -\frac{1}{3}e^{-x}\cos(3x) - \int -e^{-x}\cdot -\frac{1}{3}\cos(3x)\,dx$$

$$= -\frac{1}{3}e^{-x}\cos(3x) - \int \underset{u}{e^{-x}}\,\underset{v'}{\frac{1}{3}\cos(3x)}\,dx$$

$u = e^{-x}$ $u' = -e^{-x}$

$v' = \frac{1}{3}\cos(3x)$ $v = \frac{1}{9}\sin(3x)$

$$\int e^{-x}\sin(3x)\,dx = -\frac{1}{3}e^{-x}\cos(3x) - e^{-x}\frac{1}{9}\sin(3x) + \int -e^{-x}\frac{1}{9}\sin(3x)\,dx$$

$$\frac{10}{9}\int e^{-x}\sin(3x) = -\frac{1}{3}e^{-x}\cos(3x) - \frac{1}{9}e^{-x}\sin(3x)$$

$$\int e^{-x}\sin(x) = -\frac{3}{10}e^{-x}\cos(3x) - \frac{1}{10}e^{-x}\sin(3x) + c$$

(7) Find $\int_0^{\pi/2} \cos^3 x \sin x \, dx$.

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$$\int_1^0 u^3 \sin x \frac{dx}{du} \, du = \int_1^0 u^3 \sin x \frac{1}{-\sin x} \, du$$

$$= \int_0^1 u^3 \, du = \left[\frac{1}{4}u^4\right]_0^1 = \frac{1}{4}$$

(8) Find $\int \cos(6x)\cos(2x)\,dx$.

$\cos(A+B) = \cos(A)\cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$

$= \frac{1}{2}\int \cos(8x) + \cos(4x)\,dx = \frac{1}{16}\sin(8x) + \frac{1}{8}\sin(4x) + C$

(9) Find $\int \frac{1}{1+9x^2}\, dx.$ $= \int \frac{1}{1+(3x)^2} dx$

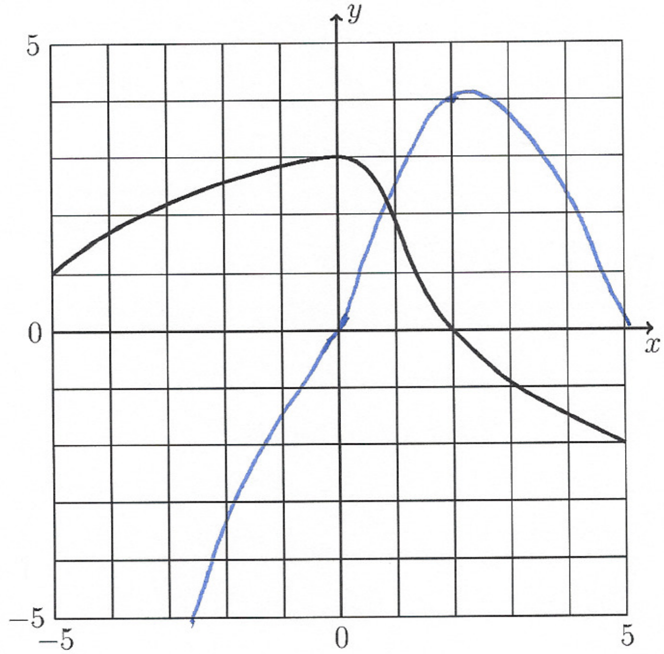
$u = 3x$

$\frac{du}{dx} = 3$

$\int \frac{1}{1+u^2} \cdot \frac{dx}{du}\, du = \int \frac{1}{1+u^2} \cdot \frac{1}{3}\, du$

$= \frac{1}{3} \tan^{-1}(u) + c = \frac{1}{3} \tan^{-1}(3x) + c$

(10) Consider the graph of the function f drawn below.



Let $g(x) = \int_0^x f(t)\, dt$.

Sketch $g(x)$, and find $g(0)$, $g'(0)$ and $g'(2)$.

$g(0)=0$

$g'(0)=3$

$g'(2)=0$