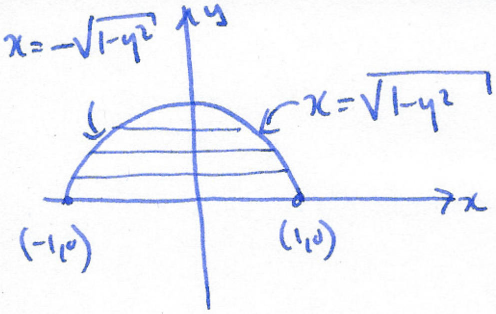


$$\int_{0}^{1}\int_{-\sqrt{1-y^2}}^{+\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

biggest value of y → 1

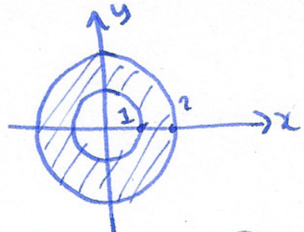
$+\sqrt{1-y^2}$ ← right boundary

smallest value of y → 0

$-\sqrt{1-y^2}$ ← left boundary

§15.2 Double integrals over general regions

Problem

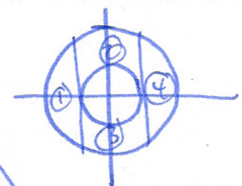


boundary of region not function of x or y.

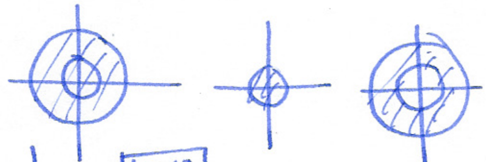
observation if a region D is the union of two disjoint regions D_1, D_2

then $\iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)\,dA$

solution ① chop region into nice regions

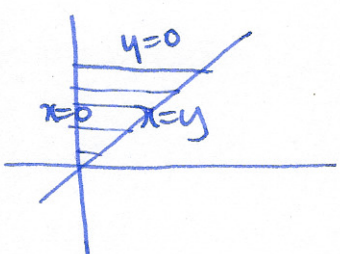


② write D as $D_1 \setminus D_2$

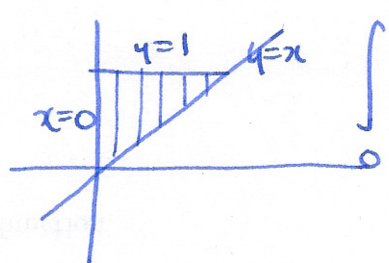


$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\,\cancel{dA}\,dy\,dx - \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\,\cancel{dA}\,dy\,dx$$

Changing order of integration

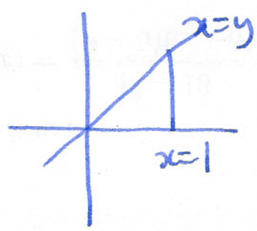


$$\int_0^1\int_0^y f(x,y)\,dx\,dy$$



$$\int_0^1\int_x^1 f(x,y)\,dy\,dx$$

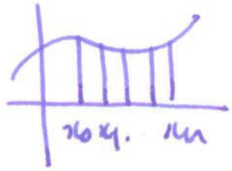
Example $\displaystyle\int_0^1\int_y^1 \frac{\sin x}{x}\,dx\,dy \xrightarrow{\text{draw region}}$



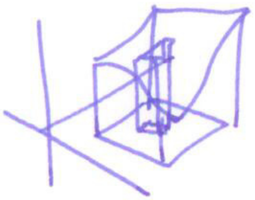
$\xrightarrow{\text{new limits}} \displaystyle\int_0^1\int_0^x \frac{\sin x}{x}\,dy\,dx$

can now do integral...

Intuition



$$\sum_{i=1}^{n} f(x_i)\Delta x_i \longrightarrow \int_a^b f(x)\,dx$$



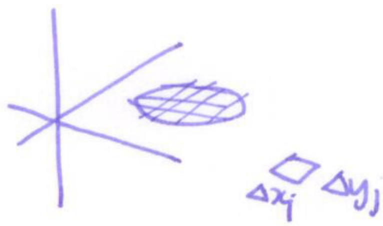
$$\sum_i \sum_j f(x_i,y_i)\,\Delta x_i \Delta y_i \longrightarrow \int_a^b \int_c^d f(x,y)\,dx\,dy$$

§15.3 Triple integrals

1 var

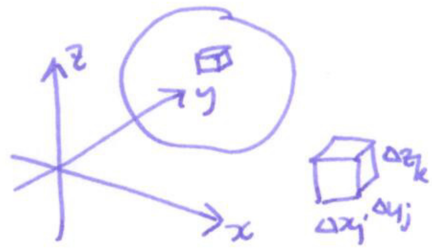
$\sum f(x_i)\Delta x_i$

2 var



$\sum f(x_i,y_i)\Delta x_i \Delta y_j$

3 var



$\sum f(x_i,y_j,z_k)\Delta x_i \Delta y_j \Delta z_k$

notation $\iiint_R f(x,y,z)\,dV$

Example integrate $f(x,y,z) = x^2 e^{y+3z}$ over the box $\underset{x}{[1,2]}\times\underset{y}{[3,4]}\times\underset{z}{[0,1]}$

$$\int_0^1 \int_3^4 \int_1^2 x^2 e^{y+3z}\,dx\,dy\,dz$$

inner integral: $\int_1^2 x^2 e^{y+3z}\,dx = e^{y+3z}\int_1^2 x^2\,dx = e^{y+3z}\left[\frac{1}{3}x^3\right]_1^2 = \frac{7}{3}e^{y+3z}$

$\int_0^1\int_3^4 \frac{7}{3}e^{y+3z}\,dy\,dz$ inner integral: $\int_3^4 \frac{7}{3}e^{y}e^{3z}\,dy = \frac{7}{3}e^{3z}\left[e^y\right]_3^4 = \frac{7}{3}(e^4-e^3)e^{3z}$

$$\int_0^1 \frac{7}{3}(e^4-e^3)e^{3z}\,dz = \frac{7}{3}(e^4-e^3)\int_0^1 e^{3z}\,dz = \frac{7}{3}(e^4-e^3)\left[e^{3z}\cdot\frac{1}{3}\right]_0^1$$

$$= \frac{7}{3}(e^4-e^3)\frac{1}{3}(e^3-1)$$