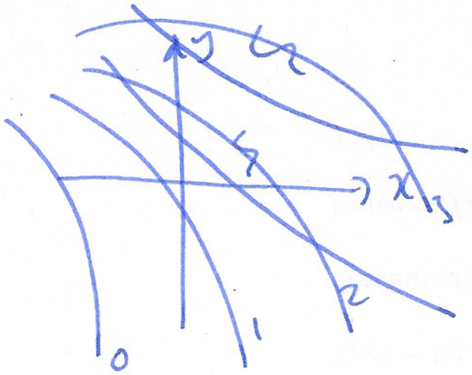
f_{xy} = " rate of change of f_x in y-direction" } equal if f_{xy} cts.
f_{yx} = " rate of change of f_y in x-direction" }

warning : f_x, f_y exist $\not\Rightarrow$ f cts !

Interpreting contour maps | level sets of $f(x,y)$.



$f_x > 0$ (numbers on contours increasing)
$f_{xx} < 0$ (contour lines getting further apart)
$f_y > 0$ (numbers on contour lines increasing)
$f_{yy} > 0$ (contour lines getting closer together)
$f_{xy} > 0$ (contours get closer together in x-direction as we move in y-direction)

Examples ① $f(x,y) = x\sin(x+y)$

$f_x = \sin(x+y) + x\cos(x+y)$

$f_y = x\cos(x+y)$

② $f(x,y) = \dfrac{ae^{xy}}{y}$ $\qquad f_x = \dfrac{aye^{xy}}{y}$ $\qquad f_y = \dfrac{yaye^{xy} - ae^{xy}}{y^2}$

Functions of 3 vars $f: \mathbb{R}^3 \to \mathbb{R}$ $f(x,y,z)$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ "diff wrt z, keeping x,y fixed" etc.

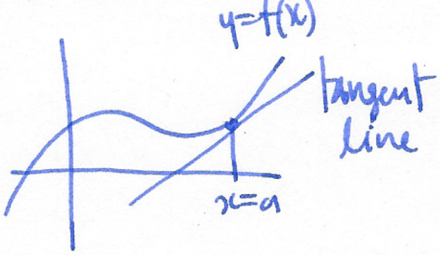
Example $f(x,y) = xy + yz + xyz$

$f_x = y + yz$, $f_y = x + z + xz$, $f_z = y + xy$

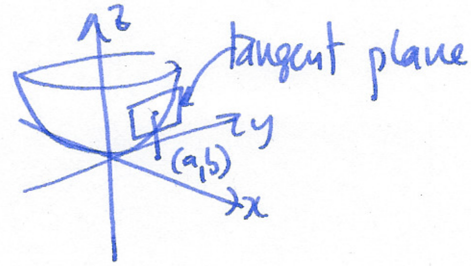
Thm If 2nd order derivatives are cts, then mixed partials are equal, i.e. $f_{xy} = f_{yx}$ $\quad f_{xz} = f_{zx}$ etc.

§14.4 Differentiability, linear approximations and tangent planes

$f: \mathbb{R} \to \mathbb{R}$



$f: \mathbb{R}^2 \to \mathbb{R}$



linear approximation at $x=a$ is $L(x) = f(a) + f'(a)(x-a)$

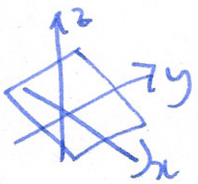
tangent line is $y = L(x)$

linear approximation at $(x,y) = (a,b)$ is

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

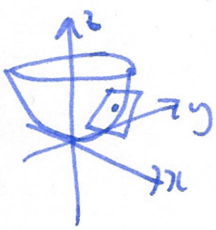
tangent plane is $z = L(x,y)$.

<u>explanation</u> consider plane $z = cx + dy$



slope in x-direction is $\frac{\partial z}{\partial x} = c$

slope in y-direction is $\frac{\partial z}{\partial y} = d$



at (a,b) slope in x-direction is $\frac{\partial f}{\partial x}(a,b) = f_x(a,b)$

y-direction is $\frac{\partial f}{\partial y}(a,b) = f_y(a,b)$

<u>Example</u> find tangent plane to $f(x,y) = x^2 + y^2$ at $(1,1)$

$f(1,1) = 2 \quad \frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = 2y$ so $L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 2 + 2(x-1) + 2(y-1)$

<u>Def$^{\underline{n}}$</u> $f(x,y)$ is <u>locally linear</u> at (a,b) if $L(x,y)$ approximates $f(x,y)$ to first order, i.e.

$$f(x,y) = L(x,y) + \underbrace{\epsilon(x,y)}_{\text{any function s.t. } \epsilon(x,y) \to 0 \text{ as } (x,y) \to 0} \sqrt{(x-a)^2 + (y-b)^2}$$

for (x,y) close to (a,b)

<u>Problem</u> f_x, f_y exist $\not\Rightarrow$ f locally linear.