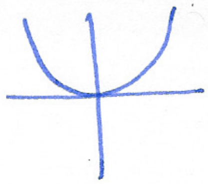
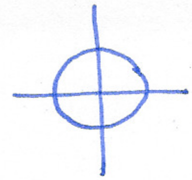
§12.6 Quadric surfaces

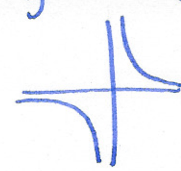
Quadratic expressions in $\mathbb{R}^2$: $y=x^2$



$x^2+y^2=1$

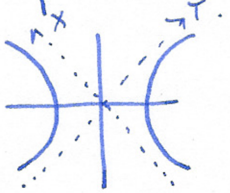


$xy=1$



$x^2-y^2=1$

$y^2=x^2-1$



change of coordinates

$X = x+y$

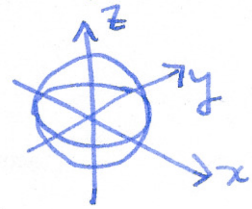
$Y = x-y$

$x^2-y^2=1 \rightsquigarrow XY=1.$

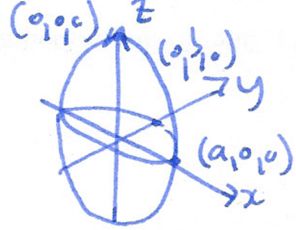
$xy=0$

A <u>quadric surface</u> is a surface defined by a quadratic expression in 3 vars.

<u>Examples</u> sphere of radius r: $x^2+y^2+z^2=r^2$



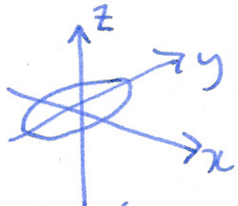
<u>ellipsoids</u> $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2=1$



<u>Q</u>: how do we work out what the surface looks like?

<u>Tool</u>: <u>traces</u> | <u>cross-sections</u>: intersections of the surface with planes parallel to the coordinate axes.

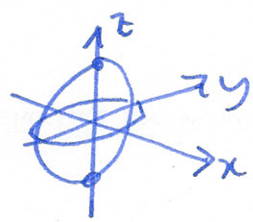
<u>Example</u> $\frac{x^2}{1}+\frac{y^2}{4}+\frac{z^2}{4}=1$



xy-trace (set $z=0$) $x^2+\frac{y^2}{4}=1$ (ellipse in xy-plane).

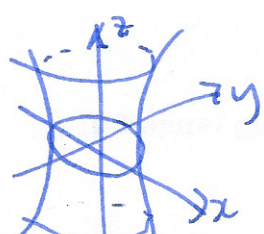
what happens when you vary z? $x^2+\frac{y^2}{4}=1-\frac{z^2}{4}$

- z large +ve no solutions $(z>4)$
- z large −ve no solution $(z<-4)$



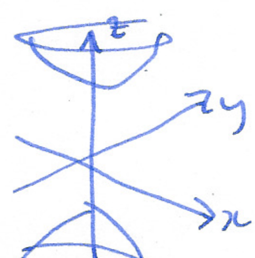
<u>hyperboloids</u> $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2-\left(\frac{z}{c}\right)^2=1$

hyperboloid of 1-sheet



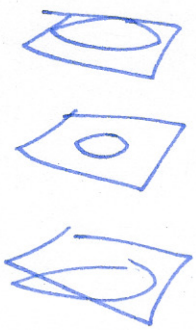
$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2-\left(\frac{z}{c}\right)^2=-1$

hyperboloid of 2-sheets.



traces parallel to the xy-plane:

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\underbrace{1+\left(\frac{z}{c}\right)^2}_{\text{always +ve, so always solutions}}$$



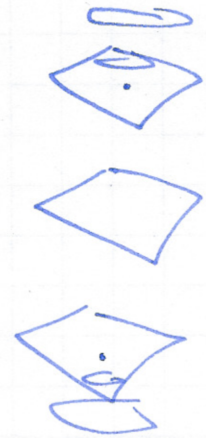
$z=1$.

$z=0$

$z=-1$

(say this hyperbola is aligned with the z-axis)

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{z}{c}\right)^2-1.$$



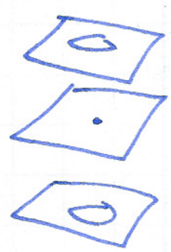
$z\geq c$ solutions

$z=0$ no solutions

$z\leq -c$ solutn

<u>elliptic cones</u> $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{z}{c}\right)^2$

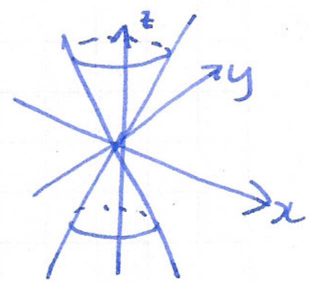
cross-sections parallel to xy-plane.



$z=1$

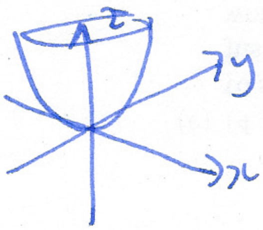
$z=0$

$z=-1$

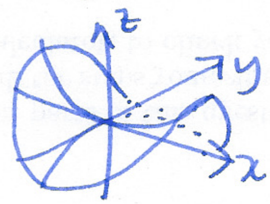


<u>paraboloids</u>

elliptic paraboloid $z=\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2$



hyperbolic paraboloid $z=\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2$.

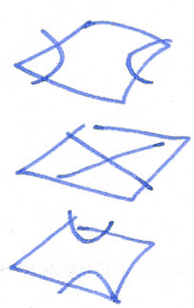


horizontal traces

$z\cancel{>0}=+1$

$z=0$

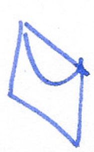
$z\cancel{<0}=-1$



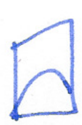
$z=0$

vertical traces

$x=c$



$y=c$



$x=c$

$y=c$