MTH 233 Calculus 3

Joseph Maher joseph@maher.org.uk
web page: http://www.math.csi.cuny.edu/~maher
office: 1S-222 office hours M: 12:20 – 2:15 W: 1:20 – 2:15.

- math tutoring 1S-214
- students with disabilities

Text: Calculus (Early transcendentals) Rogawski + Adams.

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§12.1 vectors (2d)

scalar / number

size / magnitude only

examples: 7, −4.3, etc.

examples: length
temperature
time
speed = length of velocity vector

notation: $7, \pi \in \mathbb{R}$

$s, t \in \mathbb{R}$

vector

size and direction

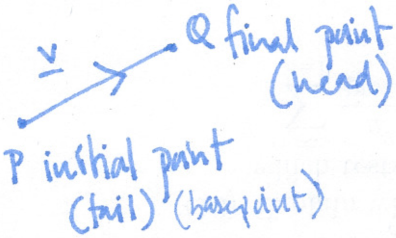
examples: 

examples: force
velocity

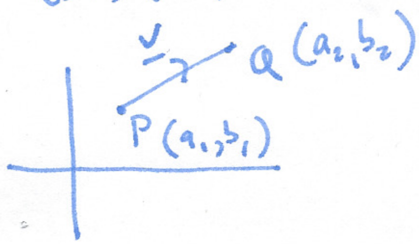
notation: $\underline{v}$ $\vec{v}$

"length 4 in direction of y-axis"

a vector $\underline{v}$ is determined by its initial and final points.



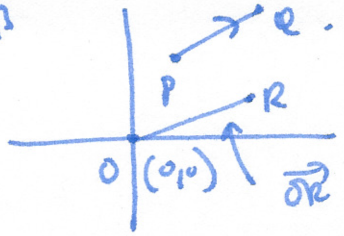
notation: $\underline{v} = \vec{v} = \overrightarrow{PQ}$



Q: how long is the vector?

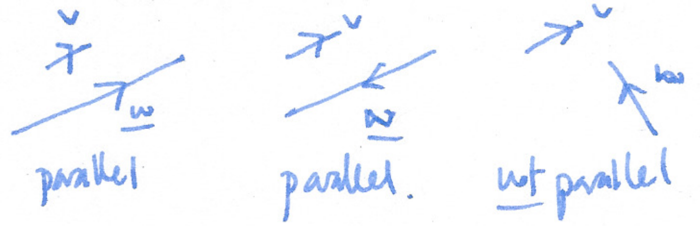
A: $\|\underline{v}\| = \|\overrightarrow{PQ}\| = \sqrt{(a_2-a_1)^2 + (b_2-b_1)^2}$.

vectors vs points

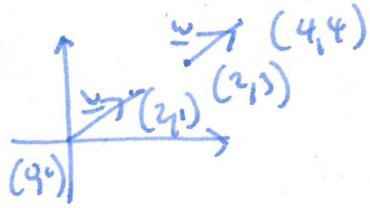


every point corresponds to a special **position vector** from $\underline{O} = (0,0)$ to R

- two vectors $\underline{v}, \underline{w}$ are **parallel** if the lines through $\underline{v}$ and $\underline{w}$ have the same slope (or same or opposite direction)

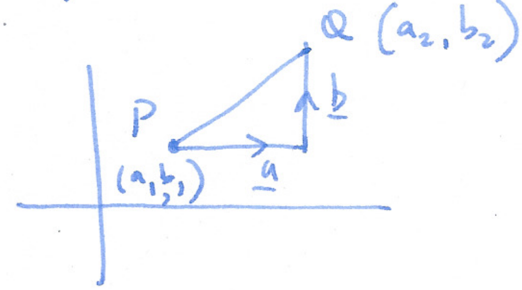


- two vectors $\underline{v}, \underline{w}$ are **translates** or **equivalent** or **equal** if they have the same length and direction.



$\underline{v} = \underline{w}$

observation every vector $\underline{v}$ is equal to a unique position vector $\underline{v}_0$ based at the origin $O = (0,0)$



Def^n^ **components** of a vector $\underline{v} = \vec{PQ}$

$P = (a_1, b_1)$ $Q = (a_2, b_2)$

are $\underline{v} = \langle a, b \rangle$ where $a = a_2 - a_1$ x-component
$b = b_2 - b_1$ y-component

observation $\|\underline{v}\| = \sqrt{a^2 + b^2}$

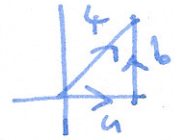
- the components $\langle a, b \rangle$ determine length and direction, so two vectors are equal if and only if they have the same components.
- $\underline{v} = \langle a, b \rangle$ does not determine the basepoint of the vector.

convention all vectors based at the origin $O = (0,0)$ unless otherwise stated.

- **special vector** $\underline{0} = \langle 0, 0 \rangle$ **zero vector** (not the origin $(0,0)$!)

(no direction!)

<u>Example</u> A vector $\underline{v}$ has length 4, lies in the first quadrant, and makes an angle of $\pi/4$ with the x-axis. Find the components of $\underline{v}$



Vector addition and scalar multiplication

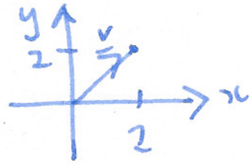
<u>scalar multiplication</u> λ scalar / number

$\underline{v}$ vector

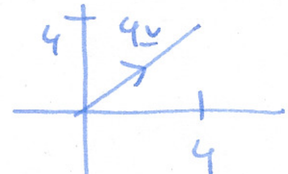
$\lambda \underline{v}$ is the vector with the same direction as $\underline{v}$ but length $\lambda \|\underline{v}\|$ if $\lambda > 0$

$\lambda \underline{v}$ " opposite " $\lambda \|\underline{v}\|$ if $\lambda < 0$

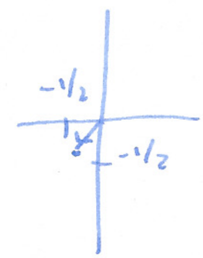
<u>Example</u> $\underline{v} = \langle 1,1 \rangle$



$4\underline{v} =$

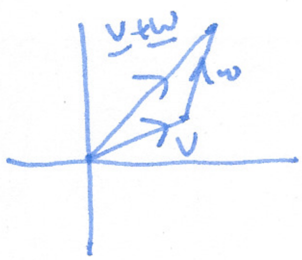


$-\frac{1}{2}\underline{v}$

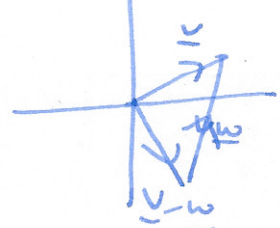


<u>observation</u> $\underline{v}$ is parallel to $\underline{w}$ iff $\underline{v} = \lambda \underline{w}$ for some λ.

<u>vector addition</u> • $\underline{v}, \underline{w}$ vectors. translate $\underline{w}$ so that the beginning of $\underline{w}$ is at the end of $\underline{v}$, then $\underline{v}+\underline{w}$ is the vector from the beginning of $\underline{v}$ to the end of $\underline{w}$



$\underline{v} - \underline{w} = \underline{v} + (-1)\underline{w}$



<u>vector operations in components</u>

If $\underline{v} = \langle a,b \rangle$ $\underline{w} = \langle c,d \rangle$

then: • $\lambda \underline{v} = \langle \lambda a, \lambda b \rangle$

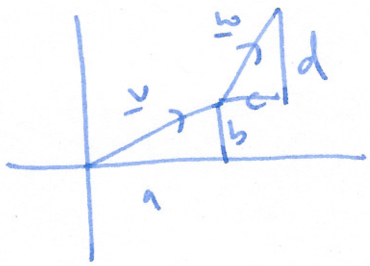
• $\underline{v}+\underline{w} = \langle a+c, b+d \rangle = \langle a,b \rangle + \langle c,d \rangle = \underline{w}+\underline{v}$

• $\underline{v}-\underline{w} = \langle a-c, b-d \rangle = \langle a,b \rangle - \langle c,d \rangle \neq \underline{w}-\underline{v}$!

• $\underline{v}+\underline{0} = \langle a,b \rangle + \langle 0,0 \rangle = \langle a,b \rangle = \underline{0}+\underline{v}$

important $\underline{v} - \underline{v} = \underline{0} = \langle 0, 0 \rangle$ zero vector not zero number.

check



useful properties: $\underline{u}, \underline{v}, \underline{w}$ vectors, λ scalar

- $\underline{u} + \underline{v} = \underline{v} + \underline{w}$ (commutative)
- $\underline{u} + (\underline{v} + \underline{w}) = (\underline{u} + \underline{v}) + \underline{w}$ (associative)
- $\lambda(\underline{v} + \underline{w}) = \lambda\underline{v} + \lambda\underline{w}$ (distributive)
- length $\|\lambda\underline{v}\| = |\lambda|\,\|\underline{v}\|$

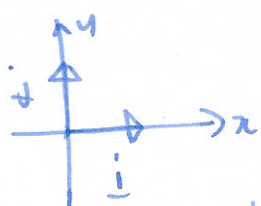
important: $\lambda + \underline{v}$ does not make sense!

unit vectors: a vector of length 1 is called a unit vector

if $\underline{v}$ is a (non-zero) vector then $\hat{\underline{v}} = \hat{\underline{e}}_{\underline{v}} = \dfrac{1}{\|\underline{v}\|}\underline{v}$ is a unit vector in the direction of $\underline{v}$.

check: $\left\| \dfrac{1}{\|\underline{v}\|}\underline{v} \right\| = \left| \dfrac{1}{\|\underline{v}\|} \right| \|\underline{v}\| = \dfrac{\|\underline{v}\|}{\|\underline{v}\|} = 1$

special vectors



$\underline{i}$ unit vector in x-direction $\underline{i} = \langle 1, 0 \rangle$
$\underline{j}$ unit vector in y-direction $\underline{j} = \langle 0, 1 \rangle$

$\underline{i}, \underline{j}$ called standard basis vectors.

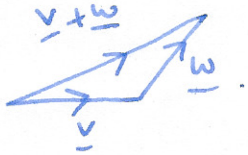
Linear combinations

$\underline{v}, \underline{w}$ vectors, r, s scalars, then $r\underline{v} + s\underline{w}$ is a linear combination of $\underline{v}$ and $\underline{w}$.

Every vector (2d) can be written as a linear combination of $\underline{i}$ and $\underline{j}$.

$\underline{v} = \langle a,b \rangle = a\langle 1,0\rangle + b\langle 0,1\rangle = a\,\underline{i} + b\,\underline{j}$.

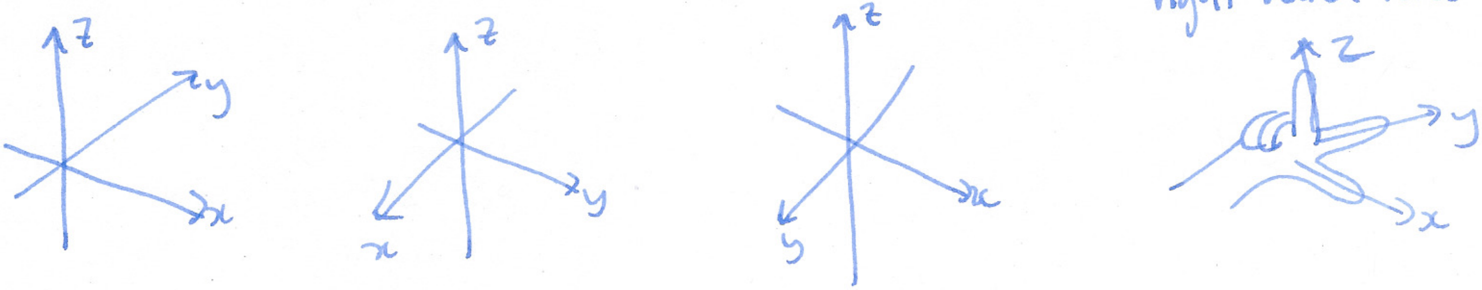
<u>triangle inequality</u>



$\|\underline{v}+\underline{w}\| \leq \|\underline{v}\| + \|\underline{w}\|$.

§ 12.2 Vectors (3d)

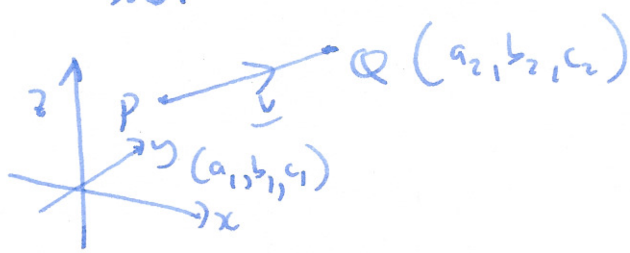
right hand rule



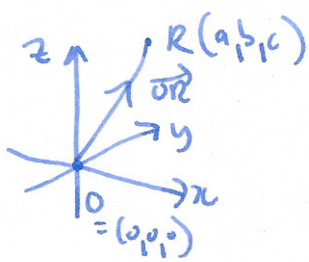
good

bad

A vector $\underline{v}$ has length and direction

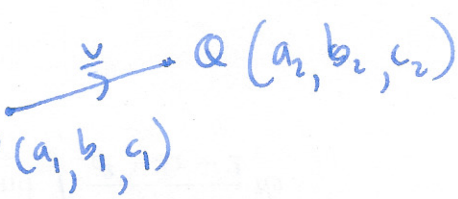


- $\underline{v}$ is determined by initial and final points $\underline{v} = \overrightarrow{PQ}$
- length: $\|\underline{v}\| = \|\overrightarrow{PQ}\| = \sqrt{(a_2-a_1)^2 + (b_2-b_1)^2 + (c_2-c_1)^2}$.
- translation: move $\underline{v}$ without changing length or direction
- equal: $\underline{v},\underline{w}$ are <u>equal</u> if they have same length and direction i.e. they are translates of each other.
- position vectors:



$\overrightarrow{OR} = \langle a,b,c\rangle$
position vector

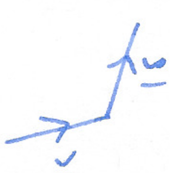
- <u>components</u>



$\underline{v} = \langle a_2-a_1, b_2-b_1, c_2-c_1\rangle = \overrightarrow{PQ}$

x-component, y-component, z-component.

- <u>vector addition</u>



$\underline{v} = \langle v_1, v_2, v_3\rangle \qquad \underline{w} = \langle w_1, w_2, w_3\rangle$

$\underline{v}+\underline{v} = \langle v_1+w_1,\ v_2+w_2,\ v_3+w_3\rangle$