Math 233 Calculus 3 Fall 18 Sample Midterm 2

(1) Find the critical points of the following functions, and use the second derivative test to classify them, if possible.

(a)

$$f(x, y) = x^3 + 2y^3 - xy$$

(b)

$$f(x, y) = e^{x+y} - xe^{2y}$$

(c)

$$f(x, y) = y \ln(x + y)$$

(2) Find the extreme values of $f(x, y) = x^3 - xy - y^2 + y$ on the square $0 \leq x \leq 1, 0 \leq y \leq 1$.

(3) Use Lagrange multipliers to find the minimum value of xy on the line $y = 4 + 5x$.

(4) Use Lagrange multipliers to find the dimensions of the cylindrical tin can with only one circular end of volume V with least surface area.

(5) Integrate the function $f(x, y) = xy$ over the triangle in the xy-plane with vertices $(2, 0), (4, 0)$ and $(3, 2)$.

(6) Evaluate the following integral by changing the order of integration:

$$\int_0^9 \int_0^{\sqrt{y}} \frac{x}{\sqrt{x^2 + y}} \, dxdy$$

(7) Write down limits for an integral over the tetrahedron with vertices $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$ and $(1, 1, 1)$.

(8) Write down limits for the following integrals.

(a) The integral of the region inside the circle $x^2 + (y - 1)^2 = 1$ and outside $x^2 + y^2 = 1$.

(b) The integral over the region in the octant $x \leq 0, y \leq 0, z \geq 0$ inside the cylinder $x^2 + y^2 = 4$ and the ellipsoid $x^2 + y^2 + 4z^2 = 16$.

(c) The integral over region with $y \geq 0$, which lies below the negative cone $z^2 = 4x^2 + 4y^2$ with $z \leq 0$, and inside the sphere of radius 6.

(9) Find the volume of the solid contained in the cylinder $x^2 + y^2 = 4$, below the surface $z = (x + y)^2 + 8$ and above the surface $z = 2xy$.

(10) Use spherical coordinates to evaluate the following integral.

$$\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{0}^{\sqrt{16-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}} dzdydx$$

(11) Let $f(x, y, z) = e^z + xy$. Evaluate

$$\int_C f ds,$$

where C is the straight line path from $(-1, 2, -2)$ to $(3, 5, 4)$.

(12) Show that the vector field $\mathbf{F} = \langle y, x^2, -z \rangle$ is not conservative. Evaluate

$$\int_C \mathbf{F.ds}$$

where C is the circle of radius 2 in the plane $y = 2$ centered on the y-axis.

(13) Show that the vector field $\mathbf{F} = \langle y \cos(xy), ze^y + x \cos(xy), e^y \rangle$ is conservative, and find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$. Evaluate

$$\int_C \mathbf{F.ds}$$

where C is the curve formed by the intersection of the plane $z = 2x+4y$ with the sphere of radius 16 in the positive octant, oriented anticlockwise around the z-axis.

(14) Write down a parameterization for the surface S consisting of the part of the sphere of radius 3 inside the positive octant, and also inside the cylinder $x^2 + y^2 = 2$.
(a) Integrate the scalar function $f(x, y, z) = (x^2 + y^2)z$ over S.
(b) Integrate the vector field $\mathbf{F}(\mathbf{x}) = \langle x, 2y, 3z \rangle$ over S.