Math 233 Calculus 3 Fall 18 Midterm 1b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find an expression for the angle between the two vectors $\langle 2, 1, -3\rangle$ and $\langle 4, -2, 1\rangle$.
(You can leave your answer in terms of trigonometric functions, you don't need to find the answer as a decimal.)

$\underline{u} \cdot \underline{v} = \|\underline{u}\| \, \|\underline{v}\| \cos\theta$ $\qquad$ $\theta = \cos^{-1}\left(\dfrac{\underline{u}\cdot\underline{v}}{\|\underline{u}\|\|\underline{v}\|}\right)$

$$\theta = \cos^{-1}\left(\frac{8-2-3}{\sqrt{14}\cdot\sqrt{21}}\right) = \cos^{-1}\left(\frac{3}{7\sqrt{6}}\right)$$

(2) (10 points) Find the equation of the plane through the point $(3, -2, 2)$ which is perpendicular to the line $\mathbf{r}(t) = \langle 1 - 2t, 2t - 1, t + 2 \rangle$.

$\underline{n} = \langle -2, 2, 1 \rangle$

$(\underline{x} - \underline{p}) \cdot \underline{n} = 0$

$(\langle x, y, z \rangle - \langle 3, -2, 2 \rangle) \cdot \langle -2, 2, 1 \rangle = 0$

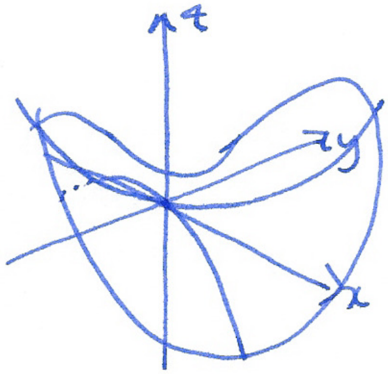
$-2x + 2y + z = -8$

(3) (10 points) Sketch the graphs of the following functions, and their contour maps/level sets.

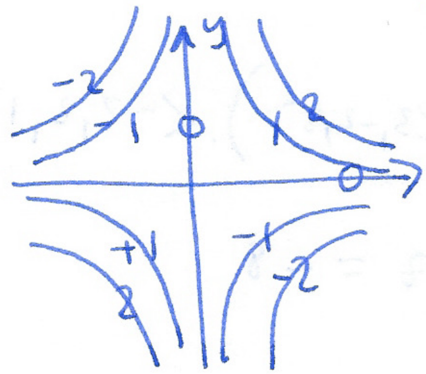
(a) $f(x, y) = xy$

(b) $f(x, y) = 4x^2 - y^2$

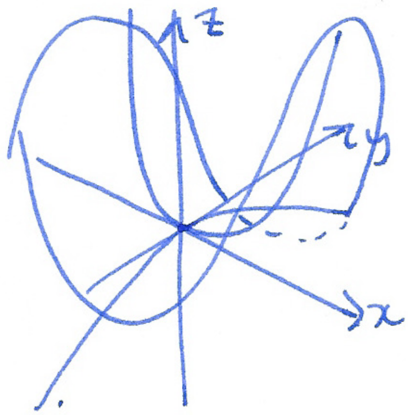
a)



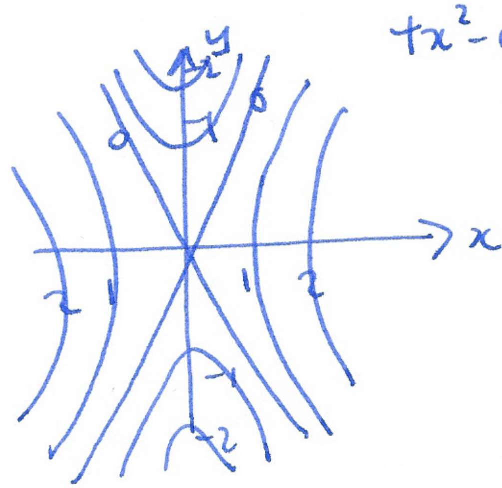
$xy = c$



b)



$4x^2 - y^2 = c.$



(4) (10 points) Write down a parameterization for the straight line segment from $(-2, 2, 1)$ to $(3, -2, 2)$. Use the integral formula for arc length to find the length of this line.

$\underline{r}(t) = \langle -2, 2, 1 \rangle + t \langle 5, -4, +1 \rangle \qquad 0 \leq t \leq 1$

$\underline{r}'(t) = \langle 5, -4, 1 \rangle$

$\|\underline{r}'(t)\| = \sqrt{42}$

arc length $= \int_0^1 \|\underline{r}'(t)\|\, dt = \int_0^1 \sqrt{42}\, dt = \left[\sqrt{42}\, t\right]_0^1 = \sqrt{42}.$

(5) (10 points) An object is thrown from the origin with initial velocity $\langle -10, 20, 10\rangle$ m/s. Find an expression for the position of the object at time t it moves under constant gravitational acceleration $\langle 0, 0, -g\rangle$ m/s^2. Feel free to take $g = 10$.

$\underline{r}''(t) = \langle 0, 0, -10\rangle$

$\underline{r}'(t) = \langle 0, 0, -10\rangle t + \underline{v}_0 = \langle 0, 0, -10\rangle t + \langle -10, 20, 10\rangle$

$\underline{r}(t) = \langle 0, 0, -10\rangle \frac{1}{2}t^2 + \langle -10, 20, 10\rangle t + \underline{r}_0$

$\underline{r}(t) = \langle 0, 0, -5\rangle t^2 + \langle -10, 20, 10\rangle t$

(6) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{y^2 - x^2}{x^2 + y^2}$$

x-axis: $\lim\limits_{(x,0)\to(0,0)} \frac{-x^2}{x^2} = \lim\limits_{x\to 0} -1 = -1$

y-axis $\lim\limits_{(0,y)\to(0,0)} \frac{y^2}{y^2} = \lim\limits_{y\to 0} 1 = 1 \neq -1 \Rightarrow \lim\limits_{(x,y)\to(0,0)} \frac{y^2-x^2}{x^2+y^2}$ DNE.

(7) Find f_{xz} derivatives for $f(x, y, z) = e^{2xyz} + \tan^{-1}(z - 3x)$.

$$f_x = e^{2xyz} \cdot 2yz + \frac{1}{1+(z-3x)^2} \cdot (-3)$$

$$f_{xz} = e^{2xyz} \cdot 2xy \cdot 2yz + e^{2xyz} \cdot 2y + 3\left(1+(z-3x)^2\right)^{-2} \cdot 2(z-3x) \cdot 1 .$$

(8) Find the linear approximation to $f(x, y) = \ln(3x + 2\sqrt{y})$ at the point $(1, 2)$.

$f(1,2) = \ln(3+2\sqrt{2})$

$f_x = \frac{1}{3x+2\sqrt{y}} \cdot 3 \qquad f_x(1,2) = \frac{3}{3+2\sqrt{2}}$

$f_y = \frac{1}{3x+2\sqrt{y}} \cdot 2\frac{1}{2}y^{-1/2} \qquad f_y(1,2) = \frac{1}{(3+2\sqrt{2}).\sqrt{2}} = \frac{1}{4+3\sqrt{2}}$

$f(x,y) \simeq \ln(3+2\sqrt{2}) + \frac{3}{3+2\sqrt{2}}(x-1) + \frac{1}{4+3\sqrt{2}}(y-2)$

(9) Find the normal vector to the surface $z^2 = x^2 - xy + 2y^2$ at the point $(2, 1, 4)$.

$F(x,y,z) = x^2 - xy + 2y^2 - z^2$

$\nabla F = \langle 2x - y, -x + 4y, -2z \rangle$

$\nabla F(2,1,4) = \langle 3, 2, -8 \rangle$

(10) The strength of a magnetic field is given by $B(x,y,z) = \dfrac{1}{(\sqrt{x^2+y^2+z^2})^3}$. $= (x^2+y^2+z^2)^{-3/2}$

Use the chain rule to find the rate of change of the field with respect to time at $t = 2$ if you are travelling on the path $\mathbf{c}(t) = \langle 2t+1, t, 1-2t\rangle$.

$$\frac{d}{dt}\left(B(\underline{c}(t))\right) = \underline{\nabla}B(\underline{c}(t)) \cdot \underline{c}'(t)$$

$$\nabla B = \left\langle -\frac{3}{2}(x^2+y^2+z^2)^{-5/2} \cdot 2x, -\frac{3}{2}(x^2+y^2+z^2)^{-5/2}\, 2y, -\frac{3}{2}(x^2+y^2+z^2)^{-5/2} \cdot 2z \right\rangle$$

$$= \frac{-3\langle x, y, z\rangle}{(x^2+y^2+z^2)^{5/2}}$$

$$\underline{c}(2) = \langle 5, 2, -3\rangle$$

$$\underline{\nabla}B(\underline{c}(2)) = \frac{-3\langle 5, 2, -3\rangle}{(38)^{5/2}}$$

$$\underline{c}'(t) = \langle 2, 1, -2\rangle$$

$$\underline{\nabla}B(\underline{c}(2)) \cdot \underline{c}'(2) = \frac{-3}{(38)^{5/2}}\langle 5, 2, -3\rangle \cdot \langle 2, 1, -2\rangle = \frac{-54}{(38)^{5/2}}$$