Math 233 Calculus 3 Fall 18 Sample midterm 1

1. Let $\vec{\mathbf{u}} = \langle\, 4, -3, 2 \,\rangle$ and $\vec{\mathbf{v}} = \langle\, 1, 3, -2 \,\rangle$.
   (a) Find $||\text{proj}_{\vec{\mathbf{v}}}\vec{\mathbf{u}}||$.
   (b) Express $\vec{\mathbf{u}}$ as the sum of $\vec{\mathbf{m}} = \vec{\mathbf{u}}_{||}$ parallel to $\vec{\mathbf{v}}$, and $\vec{\mathbf{n}} = \vec{\mathbf{u}}_{\perp}$ orthogonal to $\vec{\mathbf{v}}$.

2. Consider three points $A(1, -2, -2),\ B(2, 1, -1),\ C(3, 1, 1)$.
   (a) Find the area of the triangle formed by A, B, C.
   (b) Find the equation of the plane that contains A, B, C.

3. Find the equation of the line formed by the intersection of the two planes $x + y + 2z = 4$ and $2x - y - z = 2$.

4. For each equation below, sketch the surface in $\mathbb{R}^3$ that it describes.
   (a) $4x^2 + 4y^2 + z^2 = 16$
   (b) $z = 4x^2 - 9y^2$
   (c) $9x^2 + 4z^2 = y^2 - 12$
   (d) $4x^2 - 9y^2 = 72$

5. A particle starts at location $3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ with initial velocity $i + \boldsymbol{j} - 2\boldsymbol{k}$. Its acceleration is $\boldsymbol{a}(t) = 2\boldsymbol{i} - 3t^2\boldsymbol{j} + 2t\boldsymbol{k}$. Find the location of the particle at $t = 3$.

6. A string in the shape of a helix has a height of 16cm and makes four full rotations over a circle of radius 4cm.
   (a) Find a parametrization $\boldsymbol{r}(t)$ for the string.
   (b) Compute the length of the string.

7. Show that the following limit does not exist.
$$\lim_{(x,y)\to(0,0)} \frac{x^2y - xy^2}{x^3 + y^3}$$

8. Find all the second order partial derivatives of
$$f(x, y, z) = xy\ln(y + z) + e^{yz}\tan(x + z)$$
   You may assume that mixed partials are equal.

9. Find the equation of the tangent plane to the surface $z = 3x^4 - 2y^2$ at the point $(1, 1, 1)$.

10. Find the linear approximation to the function $f(x, y, z) = e^{4xz} + \sqrt{x^2 + y^2}$ at the point $(2, -3, 1)$.

11. Find the normal vector to the surface $z^2 = 4x^2 - 7y^2$ at the point $(2, -1, 3)$.

12. You are standing on a surface given by the equation $z = 2x^2 - 4xy - 6y^2$. If you're standing at the point $(2, 1, -6)$, in which direction is the fastest way up?

13. The temperature in the solar system is given by

$$T(x, y, z) = \frac{10^5}{x^2 + y^2 + z^2}$$

If a comet travels along the path $\boldsymbol{r}(t) = (t, 2t, 9 - t^2)$, use the chain rule to determine how fast the temperature is changing when $t = 2$.

14. Find the critical points of the following functions, and use the second derivative test to classify them, if possible.

    (a) $f(x, y) = x^3 - 6xy + y^3$

    (b) $f(x, y) = e^y - 3ye^x$

    (c) $f(x, y) = 2y \ln(x + y)$

15. Find the extreme values of $f(x, y) = 4x^2 - 2y^2$ on the square $0 \leq x \leq 1, 0 \leq y \leq 1$.