Q1 a) $\text{proj}_{\underline{v}}\underline{u} = \dfrac{\underline{u}\cdot\underline{v}}{\underline{v}\cdot\underline{v}}\underline{v} = \dfrac{4-9-4}{1+9+4}\langle 1,3,-2\rangle = \dfrac{-4}{14}\langle 1,3,-2\rangle$

$\|\text{proj}_{\underline{v}}\underline{u}\| = \left\|\dfrac{-2}{7}\langle 1,3,-2\rangle\right\| = \dfrac{2}{7}\sqrt{1+9+4} = \dfrac{2\sqrt{14}}{7}$

b) $\underline{u} = \underbrace{-\tfrac{2}{7}\langle 1,3,-2\rangle}_{\underline{w}=\underline{u}_{\parallel}} + \underbrace{\left(\langle 4,-3,2\rangle + \tfrac{2}{7}\langle 1,3,-2\rangle\right)}_{\underline{n}=\underline{u}_{\perp} = \langle \frac{26}{7}, \frac{-15}{7}, \frac{10}{7}\rangle}$

Q2 a) $\vec{AB} = \langle 1,3,1\rangle \quad \vec{AC} = \langle 2,3,3\rangle$

$\text{area}(ABC) = \frac{1}{2}\|\vec{AB}\times\vec{AC}\|$

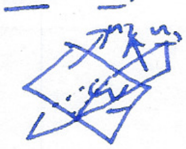
$\vec{AB}\times\vec{AC} = \begin{vmatrix} i & j & k \\ 1 & 3 & 1 \\ 2 & 3 & 3\end{vmatrix} = \langle 6,-1,-3\rangle \quad \text{area} = \frac{1}{2}\sqrt{36+1+9} = \frac{1}{2}\sqrt{46}$

b) $\underline{n} = \langle 6,-1,-3\rangle \quad \underline{P} = \langle 1,-2,-2\rangle$

$(\underline{x}-\underline{p})\cdot\underline{n} = 0 \quad (\langle 1,-2,-2\rangle + \langle x,y,z\rangle)\cdot\langle 6,-1,-3\rangle = 0$

$6x - y - 3z = 14$

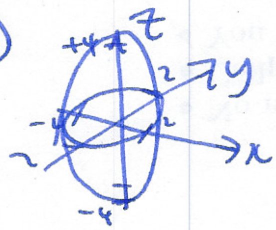
Q3 $\underline{u}_1 = \langle 1,1,2\rangle \quad \underline{u}_2 = \langle 2,-1,-1\rangle \quad \underline{u}_1\times\underline{u}_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 2 & -1 & -1\end{vmatrix} = \langle 1,5,-3\rangle = \underline{v}$



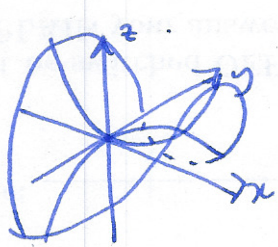
find point on line: $x+y+2z=4$, $2x-y-z=2$

try $z=0$: $\left.\begin{matrix} x+y=4 \\ 2x-y=2\end{matrix}\right\}$ $3x=6 \quad x=2, y=2 \quad L(t) = \langle 2,2,0\rangle + t\langle 1,5,-3\rangle$.

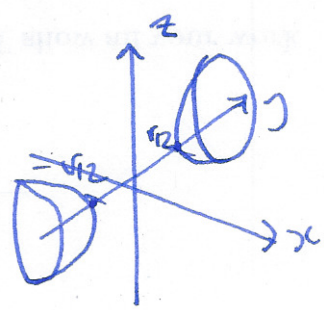
Q4 a)



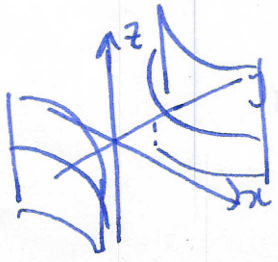
b)



c)



d)



Q5 $\underline{r}(0) = \langle 3,-2,1\rangle \quad \underline{r}'(0) = \cancel{4}\cdot\langle 1,1,-2\rangle$

$\underline{r}''(t) = \langle 2,-3t^2,2t\rangle$

$\underline{r}'(t) = \langle 2t,-t^3,t^2\rangle + \underline{r}'(0) = \langle 2t,-t^3,t^2\rangle + \langle \cancel{4},1,1,-2\rangle$

$\underline{r}(t) = \langle t^2,-\frac{1}{4}t^4,\frac{1}{3}t^3\rangle + t\langle 1,1,-2\rangle + \underline{r}(0)$

$= \langle t^2+t,-\frac{1}{4}t^4+t,\frac{1}{3}t^3-2t\rangle + \langle 3,-2,1\rangle$

$\underline{r}(3) = \langle 12,-\frac{81}{4}+3,3\rangle + \langle 3,-2,1\rangle = \langle 15,-\frac{85}{4},4\rangle$

Q6 a)

$\underline{r}(t) = \langle 4\cos t, 4\sin t, \frac{2t}{\pi}\rangle \quad 0 \le t \le 4\times 2\pi$

b) length $= \int_0^{8\pi} \|r'(t)\|\,dt \qquad r'(t) = \langle -4\sin t, 4\cos t, \frac{2}{\pi}\rangle$

$\|r'(t)\| = \sqrt{16\sin^2 t + 16\cos^2 t + \frac{4}{\pi^2}} = \sqrt{16+4/\pi^2}$

$\int_0^8 \sqrt{16+\frac{4}{\pi^2}}\,dt = \left[\sqrt{16+\frac{4}{\pi^2}}\,t\right]_0^8 = 8\sqrt{16+4/\pi^2}$

Q7 $\lim_{(t,0)\to(0,0)} \frac{0}{t^3} = 0 \qquad \lim_{(t,2t)\to(0,0)} \frac{2t^3-4t^3}{t^3+8t^3} = \lim_{t\to 0} \frac{-2}{9} = \frac{-2}{9} \neq 0$

so $\lim_{(x,y)\to(0,0)} \frac{x^2y-yx^2}{x^3+y^2}$ DNE.

Q8 $f_{\cancel{x}} = xy\ln(y+z) + e^{yz}\tan(x+z)$

$f_x = y\ln(y+z) + e^{yz}\sec^2(x+z)$

$f_y = x\ln(y+z) + \frac{xy}{y+z} + ze^{yz}\tan(x+z)$

$f_z = \frac{xy}{y+z} + ye^{yz}\tan(x+z) + e^{yz}\sec^2(x+z)$

$f_{xx} = e^{yz}\,2\sec(x+z)\cdot\sec(x+z)\tan(x+z)$

$f_{xy} = \ln(y+z) + \frac{y}{y+z} + ze^{yz}\sec^2(x+z)$

$$f_{xz} = \frac{y}{y+z} + ye^{yz}\sec^2(x+z) + e^{yz}\,2\sec(x+z)\sec(x+z)\tan(x+z)$$

$$f_{yy} = \frac{x}{y+z} + \frac{(y+z)x - xy}{(y+z)^2} + z^2e^{yz}\tan(x+z)$$

$$f_{yz} = \frac{x}{y+z} + \cancel{\tfrac{x}{y+z}} - xy(y+z)^{-2} + e^{yz}\tan(x+z) + zye^{zz}\tan(x+z) + ze^{yz}\sec^2(y+z)$$

$$f_{zz} = -xy(y+z)^{-2} + y^2e^{yz}\tan(x+z) + ye^{yz}\sec^2(x+z) + ye^{yz}\sec^2(x+z) + e^{yz}\,2\sec^2(x+z)\tan(x+z)$$

Q9 $\frac{\partial z}{\partial x} = 12x^3 \quad \frac{\partial z}{\partial y} = -4y \qquad \frac{\partial z}{\partial x}(1,1) = 12 \quad \frac{\partial z}{\partial y}(1,1) = -4$

tangent plane: $1 + 12(x-1) + (-4)(y-1)$.

Q10 $f(x,y,z) = e^{4xz} + \sqrt{x^2+y^2} \qquad f_x = 4ze^{4xz} + \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2x$

$f_y = \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2y$

$f_z = 4xe^{4xz}$

$$f(x,y,z) \approx f(2,-3,1) + f_x(2,-3,1)(x-2) + f_y(2,-3,1)(y+3) + f_z(2,-3,1)(z-1)$$

$$\approx e^8 + \sqrt{13} + \left(4e^8 + \frac{2}{\sqrt{13}}\right)(x-2) + \left(\frac{-3}{\sqrt{13}}\right)(y+3) + (8e^8)(z-1).$$

Q11 $f(x,y,z) = 4x^2 - 7y^2 - z^2 \quad \nabla f = \langle 8x, -14y, -2z\rangle$

$\nabla f(2,-1,3) = \langle 16, 14, -6\rangle$

Q12 $f(x,y,z) = 2x^2 - 4xy - 6y^2 \quad \nabla f = \langle 4x-4y, -4x-12y\rangle$

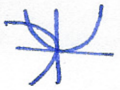
$\nabla f(2,1) = \langle 4, -20\rangle$.

Q13 $T(x,y,z) = \frac{10^5}{x^2+y^2+z^2} \quad \nabla T = 10^5\left\langle \frac{-2x}{x^2+y^2+z^2}, \frac{-2y}{x^2+y^2+z^2}, \frac{-2z}{x^2+y^2+z^2}\right\rangle$

$\underline{r}(t) = \langle t, 2t, 9-t^2\rangle \quad \underline{r}'(t) = \langle 1, 2, -2t\rangle$

$\underline{r}'(2) = \langle 1, 2, -2\rangle \quad \underline{r}(2) = \langle 2, 4, 5\rangle \quad \nabla T(\underline{r}(2)) = \langle -4, -8, -10\rangle \cdot \frac{10^5}{4+16+25}$

$$(T(r(t)))'\Big|_{t=2} = \frac{10^5}{45}\langle -4, -8, -10\rangle \cdot \langle 1, 2, -2\rangle = \frac{10^5}{45}\cdot(-4-16+20) = 0$$



Q14 a) $f(x,y) = x^3 - 6xy + y^3$.

$f_x = 3x^2 - 6y = 0 \quad 2y = x^2$

$f_y = -6x + 3y^2 = 0 \quad -2x = y^2$

$4y^2 = x^4 \quad -8x = x^4 \quad x(x^3+8) = 0 \quad x = 0, \pm 2$

↑ squaring gives sol'n up to sign $\quad y = 0, \pm 2$

$x=0, y=0 \quad x=2$, no solution $\quad x=-2, y=-2$ two solutions: $(0,0), (-2,-2)$

$f_{xx} = 6x \qquad D(0,0) = 0 - (-6)^2 = -36$ saddle.

$f_{xy} = -6$

$f_{yy} = 6y \qquad D(-2,-2) = (-12)(-12) - (-6)^2 = 144 - 36 = 108 > 0$

$f_{xx} < 0 \Rightarrow$ max.

b) $f(x,y) = e^y - 3ye^x$

$f_x = -3y = 0 \Rightarrow y = 0$

$f_y = e^y - 3e^x = 0 \Rightarrow e^x = 1/3 \quad x = -\ln(3)$

$(-\ln(3), 0)$

$f_{xx} = 0$

$f_{xy} = -3 \qquad D(-\ln(3), 0) = 0 - (-3)^2 = -9$ saddle.

$f_{yy} = e^y$

c) $f(x,y) = 2y\ln(x+y)$

$f_x = \frac{2y}{x+y} = 0 \Rightarrow y = 0$

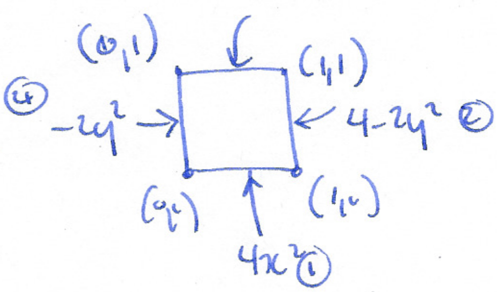
$f_y = 2\ln(x+y) + \frac{2y}{x+y} = 0 \Rightarrow 2\ln(x) = 0 \Rightarrow x = 1 \quad (1,0)$

$f_{xx} = -2y(x+y)^{-2}$

$f_{xy} = \frac{(x+y).2 - 2y}{(x+y)^2} \qquad D(1,0) = 0.(*) - (2)^2 = -4 < 0$ saddle.

$f_{yy} = \frac{2}{x+y} + \frac{(x+y).2 - 2y}{(x+y)^2}$

Q15 $f(x,y) = 4x^2 - 2y^2$



$f_x = 8x = 0$

$f_y = -4y = 0 \quad (0,0)$ not in interior of square.

① critical pt: $f_x = 8x$ at $(0,0)$

② $f_y = -4y$ at $(1,0)$

③ $f_x = 8x$ at $(0,1)$

④ $f_y = -4y$ at $(0,0)$

check these points

$f(0,0) = 0, \quad f(0,1) = -2 \quad f(1,0) = 4 \quad f(1,1) = 2$

min $\qquad$ max