**Math 233 Calculus 3 Fall 2018 Sample Final**

(1) (a) Find a vector $\mathbf{n}$ perpendicular to both the line $x = 1-t, y = 1+t, z = 1-t$ and the $x$-axis.
(b) Write down the equation of the plane through the point $(1, 2, 2)$ which is perpendicular to $\mathbf{n}$.

(2) Using the plane from the previous question, find the distance from the plane to the origin, by
(a) using the dot product.
(b) by setting up and solving a minimization problem.

(3) Let $\mathbf{a} = \langle 1, -2, 4\rangle$ and $\mathbf{b} = \langle 1, 2, -1\rangle$. Express $\mathbf{a}$ as a sum of vectors $\mathbf{x} + \mathbf{y}$, where $\mathbf{x}$ is parallel to $\mathbf{b}$, and $\mathbf{y}$ is perpendicular to $\mathbf{b}$.

(4) Sketch the level sets of the function $f(x, y, z) = x^2 + y^2 - z^2$, and calculate the gradient vector at the point $(2, 2, 1)$. Use this to find the tangent plane to $x^2 + y^2 = z^2 + 7$.

(5) Find the arc length of the curve

$$x = t^2, y = \sin t - t\cos t, z = \cos t + t\sin t$$

from $t = 0$ to $t = \pi$.

(6) A particle located at $\langle 2, -3, 7\rangle$ at $t = 0$ has velocity $\mathbf{v} = \langle -t, 3t^2, 6t^3\rangle$. Find the location of the particle at $t = 3$.

(7) You are driving anticlockwise around a circular roundabout of radius 20m, at 2m/s. When your car is facing due north, you throw a tennis ball from the car due east at 5m/s, at an angle of $\pi/6$ from horizontal. Where does the tennis ball land?

(8) Consider the function of three variables defined by

$$f(x, y, z) = e^{x-2y} + \tan(2yz).$$

(a) What is the direction of fastest rate of change of the function at the point $(2, 0, 1)$?
(b) What is the tangent plane to the surface $e^{x-2y} + \tan(2yz) = 1$ at the point $(2, 0, 1)$?

(9) Let $f(x, y) = 2x^2 - 2x + y^2 + 2$.
(a) Find the critical points of $f$ in the region $x^2 + y^2 < 9$, and use the scond derivative test to classify them.
(b) Use Lagrange multipliers to find the extreme points on the boundary $x^2 + y^2 = 9$.
(c) Use your answers above to find the extreme values of $f$ on $x^2 + y^2 \leq 9$.

(10) Find the critical points of the function $f(x, y) = 3xy - x^2y - xy^2$. Use the second derivative test to classify them, if possible.

(11) A rectangular/cuboid building loses three times as much heat per unit area from the walls than from the roof and floor. What shape should the building be to mimize heat loss if the volume is $6000\text{m}^3$?

(12) Change the order of integration to evaluate $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 y \sin(x^5)\; dx\; dy$.

(13) Evaluate the following integral

$$\int\int\int_E dV$$

where $E$ is the region in the first octant bounded by the planes $x = 1$ and $x + y + z = 4$.

(14) Write down triple integrals over the following regions.

(a) The region inside the sphere $x^2 + y^2 + x^2 = 9$ below the cone $z = \frac{1}{4}\sqrt{x^2 + y^2}$.

(b) The volume inside the cylinder $x^2 + y^2 \leq 9$, above $z = 0$ and below $2x + y + 4z = 20$.

(c) The volume of $z = 12 - 3x^2 - 3y^2$ in the positive octant.

(15) Evaluate the following integral

$$\int\int\int_E xyz\; dV$$

where $E$ lies between the spheres $\rho = 1$ and $\rho = 3$, and above the cone $\phi = \pi/3$.

(16) Integrate the vector field $\mathbf{F} = \langle y, -x, z^2 \rangle$ over the paraboloid $z = x^2 + y^2$ with $0 \leq z \leq 4$.

(17) Let $C$ be the boundary of the triangle in the plane with vertices $(0, 0)$, $(3, 0)$ and $(3, 2)$. If $\mathbf{F} = \langle \tan(1 + x^3), 2xy \rangle$, use Green's Theorem to evaluate

$$\int_C \mathbf{F}\; d\mathbf{s}.$$

(18) Let $\mathbf{F} = \langle y^2, x, z^2 \rangle$. Let $S$ be the part of the paraboloid $z = x^2 + y^2$, below the plane $z = 2$, with the upward pointing normal. Verify Stokes' Theorem in this case by directly evaluating both integrals.

(19) Let $E$ be the solid cylinder $x^2 + y^2 \leq 1$ with $0 \leq z \leq 4$, and let $\mathbf{F} = \langle x, -y, z \rangle$. Verify the divergence theorem by directly evaluating both integrals.