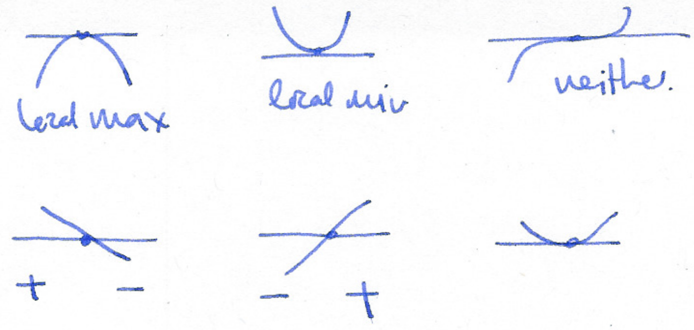
recall. critical point : $f'(x)=0$



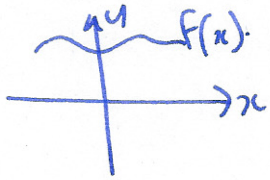
$f(x)$

$f'(x)$

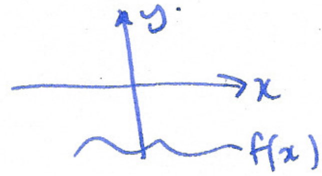
$f''(x)$ -ve +ve

§4.4 Second derivative test

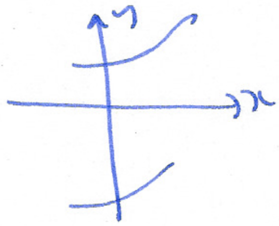
recall $f(x)>0$ f positive



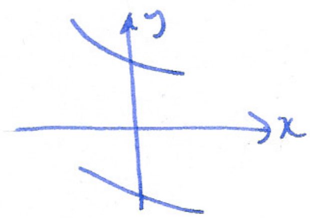
$f(x)<0$ f negative



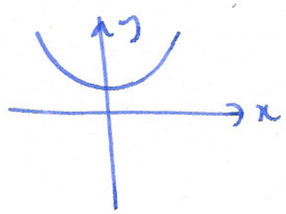
$f'(x)>0$ f increasing



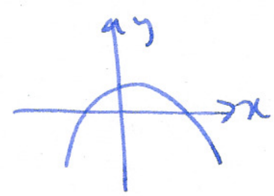
$f'(x)<0$ f decreasing



$f''(x)>0$ concave up



$f''(x)<0$ concave down



mnemonic

down up

Q: how do the slopes change?

concave up

$\leftrightarrow$ slopes increasing

$\leftrightarrow$ $f''(x)>0$

concave down

$\leftrightarrow$ slopes decreasing

$\leftrightarrow$ $f''(x)<0$

Defn Let $f(x)$ be differentiable on an interval (a,b) then

$f(x)$ is concave up $\Leftrightarrow$ $f''(x)>0$

$f(x)$ is concave down $\Leftrightarrow$ $f''(x)<0$

Defn A point of inflection is where the graph changes from concave up to concave down or vice-versa

Note : point of inflection $\underset{\not\Leftarrow}{\Rightarrow}$ $f''(x)=0$

Example $f(x) = x^3 - x = x(x^2-1) = x(x-1)(x+1)$

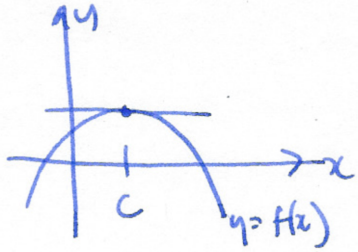
$f'(x) = 3x^2 - 1$

$f''(x) = 6x$

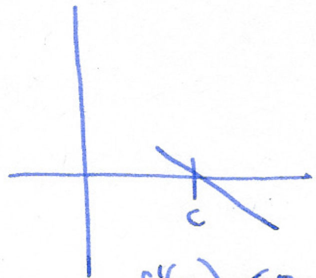
$\left.\begin{array}{l} f''(x) > 0 \text{ for } x > 0 \\ f''(x) < 0 \text{ for } x < 0 \end{array}\right\}$ inflection point at $x = 0$.

Second derivative test

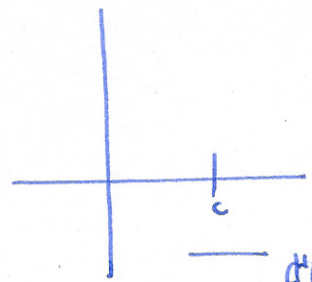
local max:



$f(x)$

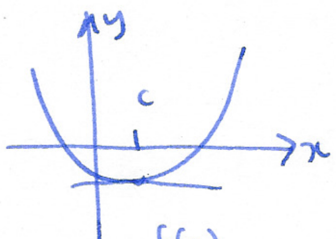


$f'(x)$

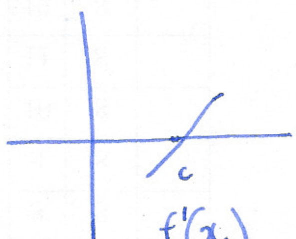


$f''(x) < 0$

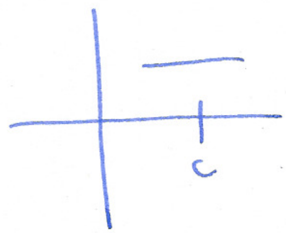
local min:



$f(x)$



$f'(x)$



$f''(x) > 0$

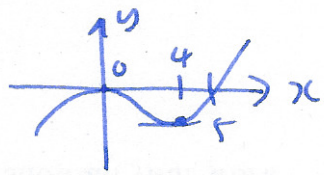
Thm suppose $f(x)$ is differentiable and c is a critical point

If $f''(c) > 0 \Rightarrow$ c is a local min ~~max~~

$f''(c) < 0 \Rightarrow$ c is a local max

$f''(c) = 0 \Rightarrow$ NO INFORMATION! (may be local max/local min/neither)

example $f(x) = x^5 - 5x^4 = x^4(x-5)$



$f'(x) = 5x^4 - 20x^3 = 5x^3(x-4)$

$f''(x) = 20x^3 - 60x^2$

critical points: $f'(x) = 0 \Rightarrow x = 0, 4$

2nd derivative test: $x = 0$ $f''(0) = 0$ no information.

$x = 4$ $f''(4) = 320 > 0 \Rightarrow$ local min

at $x = 0$ use first derivative test:

		0	
$(x-4)$	$-$		$-$
x^3	$-$		$+$
$f'(x)$	$+$		$-$

$\Rightarrow$ local max.