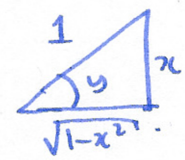
Application derivatives of inverse trig functions.

Thm $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

Proof (of $\sin^{-1}(x)$):

$$y = \sin^{-1}(x)$$
$$\sin(y) = x$$
$$\cos(y)\, y' = 1$$



$$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\sqrt{1-x^2}} \quad \square.$$

Thm $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ $\quad$ $\frac{d}{dx}\left(\cot^{-1}(x)\right) = \frac{-1}{1+x^2}$

$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}}$ $\quad$ $\frac{d}{dx}\left(\csc^{-1}(x)\right) = \frac{-1}{|x|\sqrt{x^2-1}}$

Proof (of $\tan^{-1}(x)$):

$$y = \tan^{-1}(x)$$
$$\tan(y) = x$$
$$\sec^2 y \cdot y' = 1$$
$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}. \quad \square.$$

Example $\frac{d}{dx}\left(\tan^{-1}(e^{2x})\right) = \frac{1}{1+(e^{2x})^2} \cdot e^{2x} \cdot 2$

§3.9 Derivatives of exponentials and logs

recall: $f(x) = b^x$ then $f'(x) = m_b b^x$
$f(x) = e^x$ then $f'(x) = e^x$

Thm $f(x) = b^x$ then $f'(x) = \ln(b)\, b^x$.

Proof $f(x) = b^x = e^{\ln(b)x}$ $\quad$ $f'(x) = e^{x\ln(b)} \cdot \ln(b) = b^x \cdot \ln(b)$ $\square$.

Example $f(x) = 3^{4x} = (3^4)^x$ $\therefore$ $f'(x) = \ln(3^4)(3^4)^x = 4\ln(3)\, 3^{4x}$.

Thm $f(x) = \ln(x)$ then $f'(x) = \frac{1}{x}$.