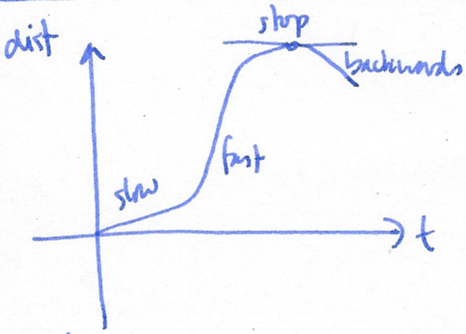
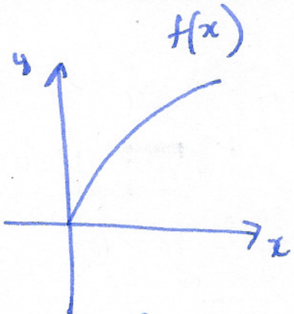
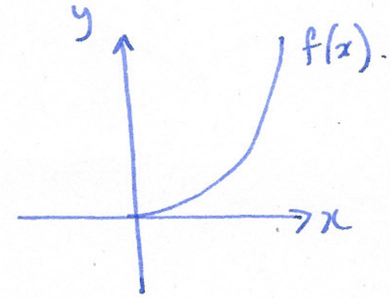
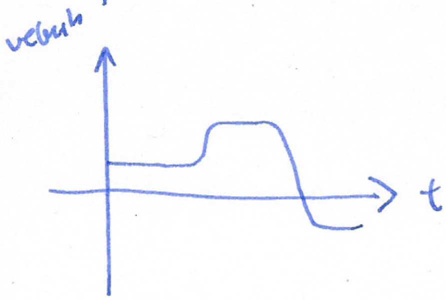
On the interpretation of graphs

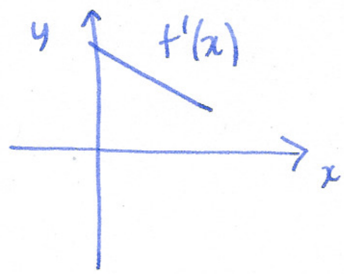


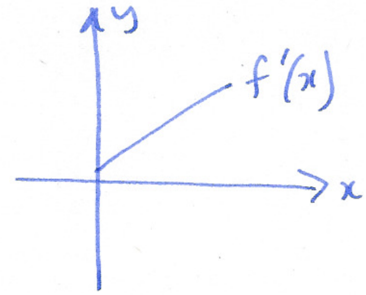


rate of change
decreasing

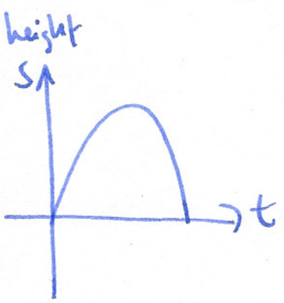








Motion under gravity



$s_0 = s(0) = $ height at $t=0$

$v_0 = v(0) = $ ~~height~~ speed at $t=0$

$g = 9.8\,m/s^2$
$= 32\,ft/s^2$

$s''(t) = v'(t) = a(t) = -g$ (constant)

$s'(t) = v(t) = -gt + v_0$

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

Q: when is the maximum height?

A: when $s'(t) = v(t) = 0$: $v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g}$ so $s\left(\frac{v_0}{g}\right) = s_0 \frac{(v_0)^2}{g} - \frac{1}{2} g \frac{v_0^2}{g^2}$

$= s_0 + \frac{1}{2}\frac{v_0^2}{g}$.

Example throw a stone upwards at 10m/s from a height of 2m. what is max height?

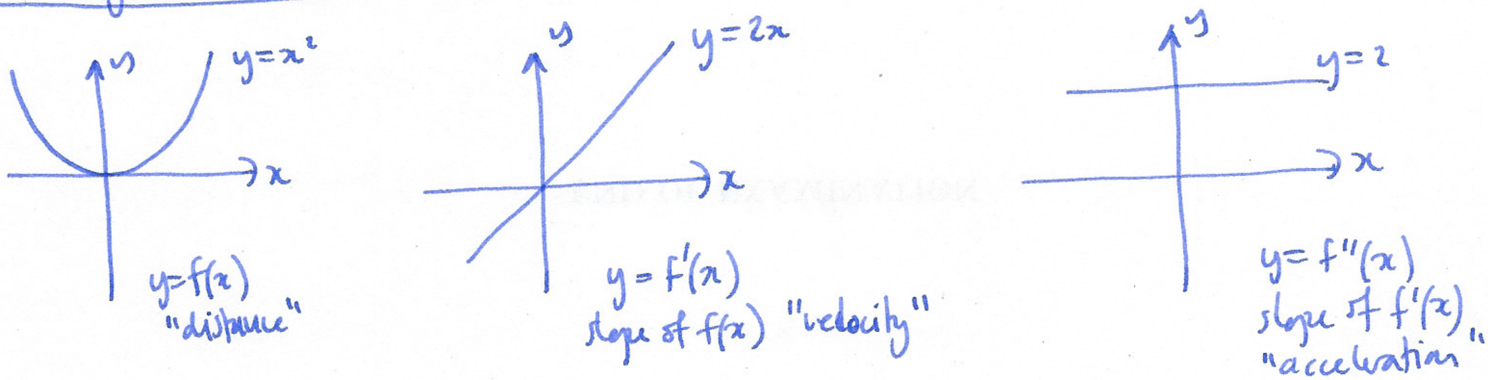
$s(t) = 2 + 10t - \frac{1}{2}gt^2$

$v(t) = 10 - gt$

$v(t) = 0 \Rightarrow t = \frac{10}{g} \simeq 1$ $\quad s(1) = 2 + 10 - 5 = 7m$.

Q: if I can throw a stone 10m high, how fast can I throw it?

§3.5 Higher derivatives



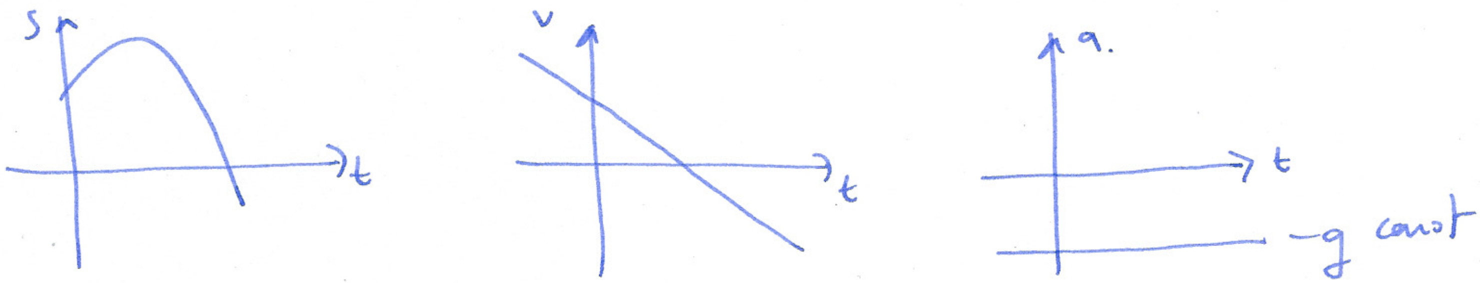
Example $f(x) = xe^x$

$$f'(x) = xe^x + e^x$$

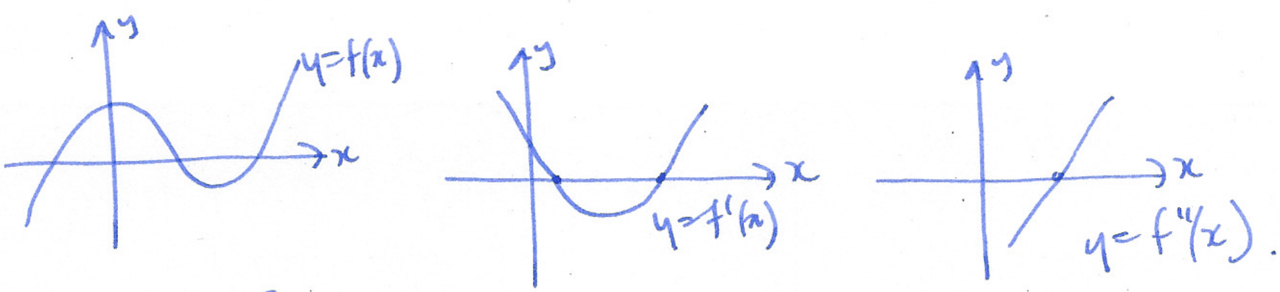
$$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$$

$$f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x \quad \text{etc.}$$

Example accelation due to gravity



Example



§3.6 Trigonametric functions

Thm $\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x.$

Proof (for $\sin x$)

$$\frac{d}{dx}(\sin(x)) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

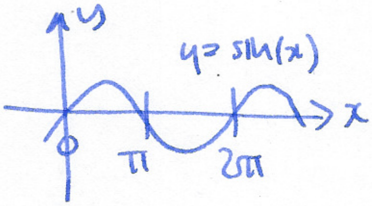
recall $\sin(A+B) = \sin A \cos B - \sin B \cos A$

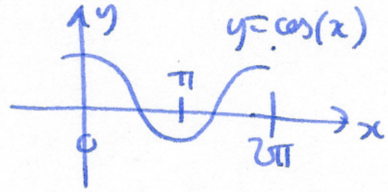
$$= \lim_{h\to 0} \frac{\sin x \cos h - \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \cdot \frac{\cos h - 1}{h} + \cos x \frac{\sin h}{h} = \sin x \cdot \underbrace{\lim_{h\to 0} \frac{\cos h - 1}{h}}_{=0} + \cos x \underbrace{\lim_{h \to 0} \frac{\cancel{\cos h}\ \sin h}{h}}_{=1}$$

$= \cos x$ □.

Q: can this be right?





Example $f(x) = x\sin(x)$

$f'(x) = x\cos(x) + \sin(x)$

Thm $\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ $\qquad$ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \tan x$.

Proof (of $\tan x$) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x}$

$= \sec^2 x$. □.

Example $\frac{d}{dx}(e^x \cos x) = e^x(-\sin x) + e^x \cos x$.

§3.7 The Chain Rule

Compositions of functions: $f(g(x)) = (f \circ g)(x)$

Examples e^{4x}, $\sin^2(x)$, etc.

Thm Chain rule If f and g are differentiable functions, then $f \circ g$ is differentiable, and $(f(g(x)))' = f'(g(x)) \cdot g'(x)$.

mnemonic: $[f(g(x))]' =$ outside'(inside). inside'.

note: $f(g(x))$

$$\mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{f} \mathbb{R}$$
$$x \longmapsto g(x) \longmapsto f(g(x))$$