MTH 231 Calculus I

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office 1S-222 office hours M 12:20–2:15
W 1:20–2:15

- math tutoring: 1S–214
- students w/disabilities

Text: Calculus, Rogawski + Adams

HW: webworks.

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§1.2 Linear and quadratic functions

recall: input set (domain) $\xrightarrow{\text{function}}$ output set (range)

examples: $f: \mathbb{R} \to \mathbb{R}$ ($\mathbb{R}$ = real numbers)
$x \mapsto x^2$ or $f(x) = x^2$ (description of function)

e.g.
$0 \mapsto 0$
$1 \mapsto 1$
$-1 \mapsto 1$
$2 \mapsto 4$ etc.

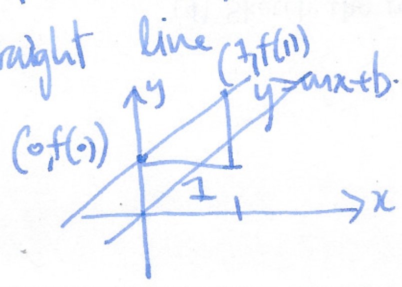
notation: $f : \mathbb{R} \longrightarrow \mathbb{R}$
name ↗ ↗ domain ↖ range.

Examples $+ : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$
$(a,b) \longmapsto a+b$

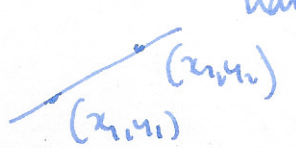
evaluation at 0: $\{$ functions $f: \mathbb{R} \to \mathbb{R} \} \longrightarrow \mathbb{R}$
$f \longmapsto f(0)$

key property: every input gives a unique output

A linear function $f: \mathbb{R} \to \mathbb{R}$ has the form $f(x) = mx + b$
(m, b "constants", i.e. don't depend on x) The graph of a linear function is a straight line

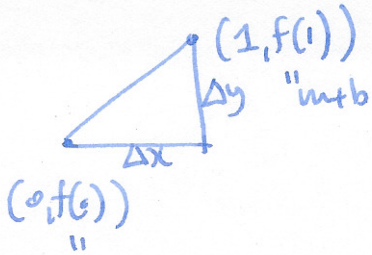


slope = $\dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{\text{rise}}{\text{run}} = \dfrac{\Delta y}{\Delta x}$



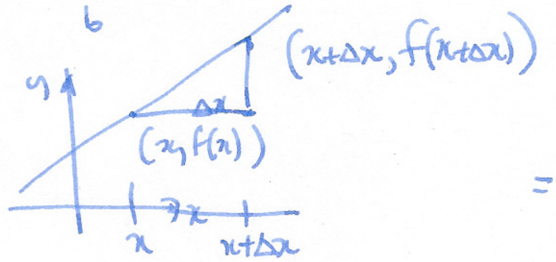
$= \dfrac{y_2 - y_1}{x_2 - x_1}$

<u>special case</u>



$$\frac{\Delta y}{\Delta x} = \frac{f(1)-f(0)}{1-0} = \frac{m+b-b}{1} = m$$

<u>general case</u>



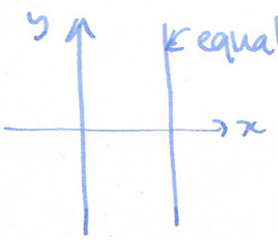
slope $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{x+\Delta x - x}$

$$= \frac{m(x+\Delta x) + b - (mx+b)}{x + \Delta x}$$

$$= \frac{\cancel{mx} + m\Delta x + \cancel{b} - \cancel{mx} - \cancel{b}}{\Delta x} = m.$$

<u>useful fact</u>: a straight line has constant slope everywhere.

<u>observations</u>
- $|m|$ large, steep slope
- $m=0$ horizontal line
- $m>0$ increasing (from left to right)
- $m<0$ decreasing
- vertical lines not graphs of functions.



equation of vertical line is $x=c$

equation vs function

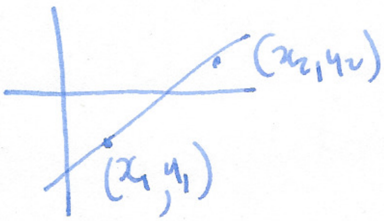
relation between variables
e.g. $x=c$
$x^2+y^2=1$

a map from one set to another e.g. $f:\mathbb{R}\to\mathbb{R}$

the graph of $f:\mathbb{R}\to\mathbb{R}$ is $y=f(x)$ (an equation!) but not every equation comes from a function.

to deal with <u>any</u> straight line use the <u>general linear equation</u> $ax+by=c$ (at least one of a,b not zero) eg: $x=c$ set $b=0, a=1$.

useful technique: find equation of line through two points.

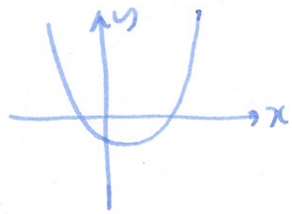


- find slope: $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- line $y - y_1 = m(x - x_1)$

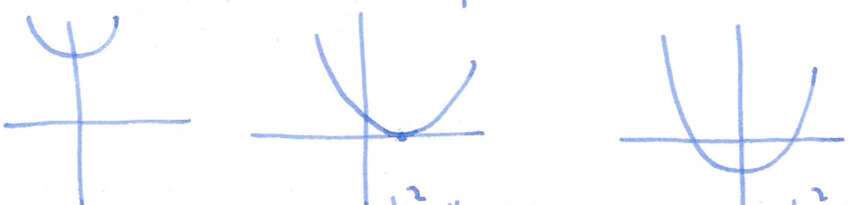
Quadratic functions

given by $f(x) = ax^2 + bx + c$ (a, b, c constants, do not depend on x)

graphs are parabolas



at most two distinct real ~~variables~~ solutions to $f(x) = 0$, given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



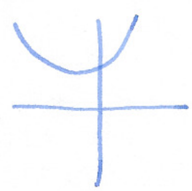
$b^2 - 4ac < 0$ $\quad$ $b^2 - 4ac = 0$ $\quad$ $b^2 - 4ac > 0$

useful techniques

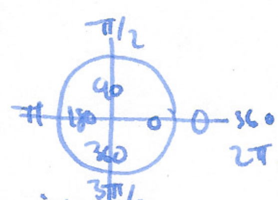
- factorization: $ax^2 + bx + c = a(x - r_1)(x - r_2)$ then r_1, r_2 are solutions/roots
- complete the square: any quadratic function can be written as

$a(x+b)^2 + c$ example:
$$x^2 + 2x + 3$$
$$(x+1)^2 + 2$$
$$x^2 + 2x + 1 + 2$$

no solutions!



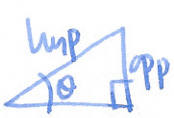
§1.4 Trig functions



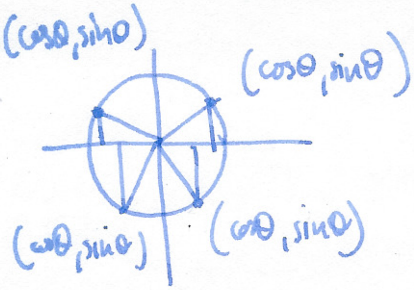
- angles vs. radians: radians win

angle in radians = distance travelled round unit circle.

- right angled triangles



$\sin\theta = \dfrac{opp}{hyp}$, $\cos\theta = \dfrac{adj}{hyp}$ } can extend these functions to be defined for all $\theta \in \mathbb{R}$.



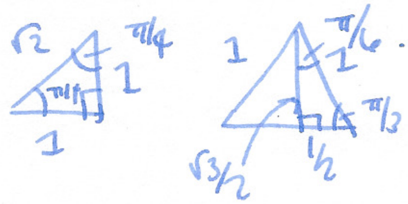
useful facts: $\sin(-\theta) = -\sin(\theta)$ (odd function)

$\cos(-\theta) = \cos(\theta)$ (even function)

special values:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

from special triangles:



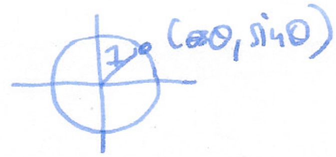
- graphs of $\sin\theta$, $\cos\theta$ are periodic with period 2π

other trig functions $\tan(x) = \frac{\text{opp}}{\text{adj}} = \frac{\sin(x)}{\cos(x)}$ $\quad \operatorname{cosec}(x) = \frac{1}{\sin(x)}$ $\quad \sec(x) = \frac{1}{\cos(x)}$

$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$

Trig identities

Pythagorean identity $\cos^2 x + \sin^2 x = 1$



$\frac{\cos^2 x + \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \leftrightarrow \cot^2 x + 1 = \operatorname{cosec}^2 x$

$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \leftrightarrow 1 + \tan^2 x = \sec^2 x$

Double angle $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

Addition: $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$