Math 231 Calculus 1 Fall 18 Midterm 3b

Name: Solutions

- I will count your best 8 out of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) (10 points) Consider the function $f(x) = \dfrac{1}{x^2 - 4x + 3}$. $= \dfrac{1}{(x-3)(x-1)}$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes $x = 1, 3$.

$\lim_{x \to \infty} \frac{1}{x^2-4x+3} = 0$ $\lim_{x \to -\infty} \frac{1}{x^2-4x+3} = 0$ horizontal asymptotes both $y=0$

b) $f'(x) = -\left((x-1)(x-3)\right)^{-2} \cdot (2x-4) = \dfrac{4-2x}{(x-1)(x-3)} = 0 \Rightarrow x = 2$

c)

	$-\infty$		1		2		∞
$-(2x-4)$		+		+	0	−	
$f'(x)$		+		−	0	−	

increasing $(-\infty, 1) \cup (1, 2)$
decreasing $(2, 3) \cup (3, \infty)$

(2) (10 points) Consider the function $f(x) = x^2 \ln(x), x > 0$.
(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them.

a) $f'(x) = 2x\ln(x) + x^2 \cdot \frac{1}{x} = 2x\ln(x) + x$

$f'(x) = 0$: $x(2\ln(x)+1) = 0$ $\quad x = 0$ ← ignore, $x > 0$
$x = e^{-1/2}$

b) $f''(x) = 2\ln(x) + 2x \cdot \frac{1}{x} + 1 = 2\ln(x) + 3$

$f''(e^{-1/2}) = -1 + 3 = 2 > 0$ local min

(3) (10 points) Find $\lim_{x\to 0} \frac{e^{2x}-1}{\sin(3x)}$. $\frac{0}{0}$ indeterminate form

l'H: $= \lim_{x\to 0} \frac{2e^{2x}}{3\cos(3x)} = \frac{2}{3}$

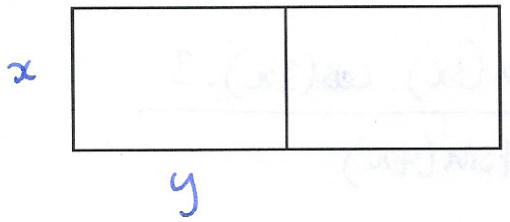
(4) (10 points) Find $\lim_{x\to 0} \frac{\sin^2(3x)}{\cos(4x)-1}$. $\frac{0}{0}$ indeterminate form

L'H: $= \lim_{x\to 0} \frac{2\sin(3x)\cdot\cos(3x)\cdot 3}{-4\sin(4x)}$ $\frac{0}{0}$ indeterminate form.

L'H: $= \lim_{x\to 0} \frac{2\cdot\cos(3x)\cdot 3\cos(3x)\cdot 3 + 2\sin(3x)\cdot(-\sin(3x)\cdot 9)}{-16\cos(4x)}$.

$= -\frac{18}{16} = -\frac{9}{8}$

(5) You wish to construct two adjacent rectangular fields of equal dimension, sharing a common fence, as shown below. If you have 360 feet of fencing, what is the largest area you can enclose?



$A = 2xy$

$L = 3x + 4y = 360$ $\qquad y = \frac{360 - 3x}{4}$

$A = \frac{2x}{3}\left(\frac{360-3x}{4}\right) = \frac{1}{6}x(360-3x) = 60x - \frac{1}{2}x^2$

$\frac{dA}{dx} = 60 - x = 0 \Rightarrow x = 60$

$A = \frac{1}{6}.60.(360-180) = 10.180 = 1800$

$3x^{1/3}$.

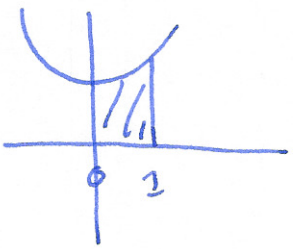
(6) (10 points) Find the indefinite integral $\int 2x^2 - 3\sqrt[3]{x}\, dx$.

$\frac{2}{3}x^3 - 3 \cdot x^{4/3} \cdot \frac{3}{4} + c$

(7) (10 points) Evaluate the indefinite integral $\int 2\cos(x) - 3e^x \, dx$.

$+2\sin(x) - 3e^x + C$

(8) (10 points) Find the area under the graph $y = 2x^4 + 3$ between $x = 0$ and $x = 1$.



$$\int_0^1 2x^4 + 3\,dx = \left[\frac{2}{5}x^5 + 3x\right]_0^1$$

$$= \frac{2}{5} + 3 = \frac{17}{5}$$

(9) (10 points) Find the indefinite integral $\int xe^{-3x^2}dx$.

$u = -3x^2$

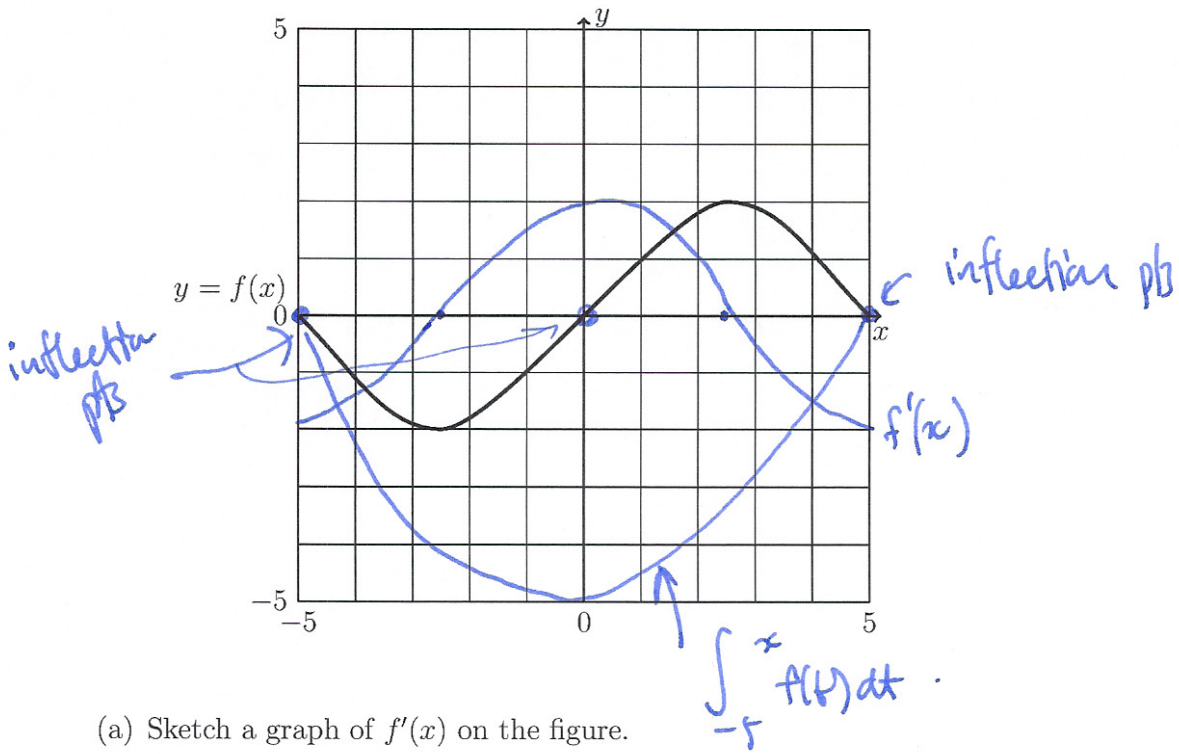
$\frac{du}{dx} = -6x$

$$\int x\, e^{u} \cdot \frac{dx}{du}\, du$$

$$= \int x\, e^{u} \frac{1}{-6x}\, du = -\frac{1}{6}\int e^{u}\, du = -\frac{1}{6}e^{u} + C$$

$$= -\frac{1}{6}e^{-3x^2} + C$$

(10) (10 points) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection.

(c) Sketch the graph of $\int_{-5}^{x} f(t)\,dt$.