Math 231 Calculus 1 Fall 18 Midterm 2b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of $f(x) = \dfrac{x}{\ln(x)}$.

$$f'(x) = \frac{\ln(x) \cdot 1 - x \cdot \frac{1}{x}}{(\ln(x))^2} = \frac{\ln(x) - 1}{(\ln(x))^2}$$

(2) (10 points) Find the derivative of the function $f(x) = e^{-2x}\sin(3x)$.

$f'(x) = -2e^{-2x}\sin(3x) + e^{-2x} \cdot \cos(3x) \cdot 3$

(3) (10 points) Find the derivative of the function $f(x) = \tan^{-1}(3\sqrt{x})$.

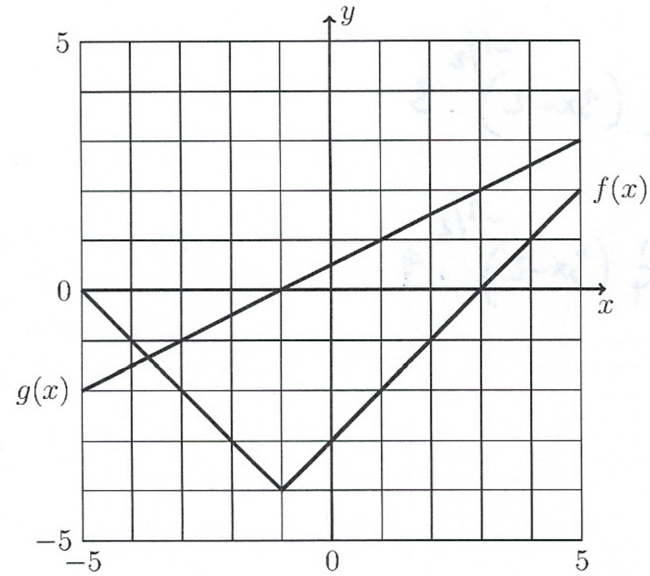
$$f'(x) = \frac{1}{1+9x} \cdot 3\,\frac{1}{2}\,x^{-1/2}$$

(4) (10 points) Find the second derivative of the function $f(x) = \sqrt{3x - 2}$.

$f'(x) = \frac{1}{2}(3x-2)^{-1/2} \cdot 3$

$f''(x) = -\frac{1}{4}(3x-2)^{-3/2} \cdot 9$

(5) (10 points) The graph of the functions f and g are shown below.



(a) Let $h(x) = f(x)/g(x)$. Find $h'(1)$.

$$h'(x) = \frac{g(x).f'(x) - f(x).g'(x)}{(g(x))^2} \qquad h'(1) = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2}$$

$$= \frac{1.1 - (-2).\frac{1}{2}}{(1)^2} = 2$$

(b) Let $h(x) = f(g(x))$. Find $h'(-1)$.

$$h'(x) = f'(g(x)).g'(x)$$

$$h'(-1) = f'(g(-1)).g'(-1) = f'(0).g'(-1) = 1.\frac{1}{2} = \frac{1}{2}$$

(6) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $x^3 - y^2 = 2xy - 4$ at the point $(2, 2)$.

$3x^2 - 2yy' = 2y + 2xy'$

$12 - 4y' = 4 + 4y'$

$8 = 8y' \qquad y' = 1$

$y - 2 = 1(x - 2)$

$y = x \cancel{+4}$

(7) (10 points) An oil tanker is leaking oil and forming a circular oil slick. If the area of the oil slick is growing at a rate of $20\text{m}^2/\text{minute}$, how fast is the radius growing when the radius is 10m? (The area of a circle is $A = \pi r^2$.)

$A(t) = \pi (r(t))^2$

$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$

$\frac{dA}{dt} = 20 \quad r = 10$:

$20 = 20\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{\pi}$ m/s

(8) (10 points) Use linear approximation to estimate $\sqrt{97}$. What is the percentage error in your approximation?

$f(x) = \sqrt{x}$ $\quad\quad$ $f(100) = 10$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad\quad$ $f'(100) = \frac{1}{20}$

$f(97) \approx f(100) + f'(100) \cdot (-3)$

$10 + \frac{1}{20} \cdot -3 = 10 - \frac{3}{20} = 9.85$

percentage error $= \dfrac{|9.85 - \sqrt{97}|}{\sqrt{97}} \times 100 \approx 0.012\%$

(9) Find the critical points for the function $f(x) = x^3 - 3x$ and use the first derivative test to classify them.

$f'(x) = 3x^2 - 3$

critical points: solve $f'(x) = 0$: $3x^2 - 3 = 0$ $\quad 3(x^2 - 1) = 0$

$3(x+1)(x-1) = 0 \quad x = \pm 1$

		-1		+1	
$x+1$	$-$		$+$		$+$
$x-1$	$-$		$-$		$+$
$f'(x)$	$+$		$-$		$+$

-1 local max

$+1$ local min

(10) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$.