Math 231 Calculus 1 Fall 18 Midterm 2a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of $f(x) = \dfrac{\ln(x)}{x}$.

$$f'(x) = \frac{x \cdot \frac{1}{x} - \ln(x) \cdot 1}{x^2} = \frac{1 - \ln(x)}{x^2}$$

(2) (10 points) Find the derivative of the function $f(x) = e^{-3x}\cos(2x)$.

$f'(x) = -3e^{-3x}\cos(2x) + e^{-3x}\cdot(-\sin(2x))\cdot 2$

(3) (10 points) Find the derivative of the function $f(x) = \tan^{-1}\left(2\sqrt{x}\right)$.

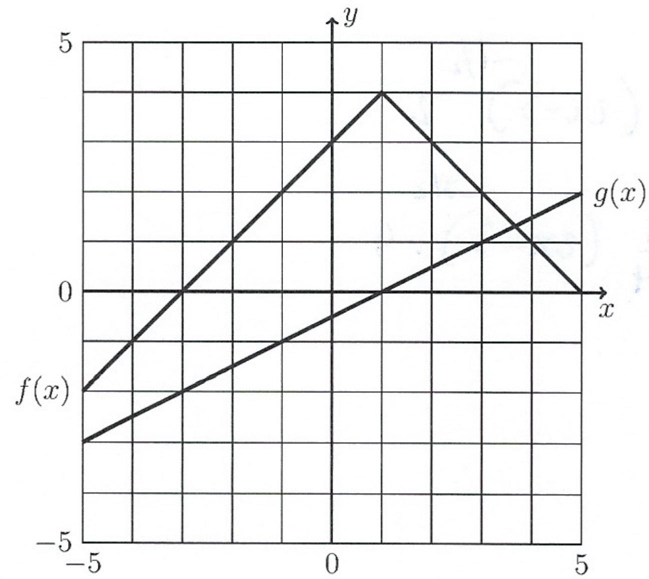
$$f'(x) = \frac{1}{1+4x} \cdot 2\,\frac{1}{2}x^{-1/2}$$

(4) (10 points) Find the second derivative of the function $f(x) = \sqrt{2x-3}$.

$f'(x) = \frac{1}{2}(2x-3)^{-1/2} \cdot 2$

$f''(x) = -\frac{1}{4}(2x-3)^{-3/2} \cdot 4$

(5) (10 points) The graph of the functions f and g are shown below.



(a) Let $h(x) = f(x)/g(x)$. Find $h'(3)$.

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} \qquad h'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2}$$

$$h'(3) = \frac{1\cdot(-1) - 2\cdot\frac{1}{2}}{(1)^2} = -2$$

(b) Let $h(x) = f(g(x))$. Find $h'(-3)$.

$$h'(x) = f'(g(x)\cdot g'(x)$$

$$h'(-3) = f'(g(-3))\cdot g'(-3) = f'(-2)\cdot\frac{1}{2} = 1\cdot\frac{1}{2} = \frac{1}{2}$$

(6) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $x^2 + y^3 = 3xy + 1$ at the point $(3, 1)$.

$2x + 3y^2 y' = 3\cancel{x}y^{\cancel{3}} + 3xy'$

$x=3, y=1$

$6 + 3y' = \cancel{3y}^{3} + 9y'$

$3 = 6y'$

$y' = 1/2$

$y - 1 = \frac{1}{2}(x-3)$

$y = \frac{1}{2}x - \frac{1}{2}$

(7) (10 points) An oil tanker is leaking oil and forming a circular oil slick. If the area of the oil slick is growing at a rate of $10\text{m}^2/\text{minute}$, how fast is the radius growing when the radius is 5m? (The area of a circle is $A = \pi r^2$.)

$A(t) = \pi(r(t))^2$

$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$

$10 = \pi \cdot 2 \cdot 5 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{\pi}$ m/s

(8) (10 points) Use linear approximation to estimate $\sqrt{98}$. What is the percentage error in your approximation?

$f(x) = \sqrt{x}$ $\qquad f(100) = 10$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\qquad f'(100) = \frac{1}{20}$

$f(98) \approx f(100) + f'(100)\cdot(-2) = 10 + \frac{1}{20}\cdot -2 = 9.9$

percentage error : $\frac{|9.9 - \sqrt{98}|}{\sqrt{98}} \times 100 \approx 0.00510\%$

(9) Find the critical points for the function $f(x) = x^3 - 12x$ and use the first derivative test to classify them.

$f'(x) = 3x^2 - 12$

solve $f'(x) = 0$: $3(x^2-4) = 0$ $3(x-2)(x+2) = 0$ $x = \pm 2$

$x-2$	$-$		$-$		$+$
$x+2$	$-$		$+$		$+$
		-2		$+2$	
$f'(x)$	$+$		$-$		$+$
		$\cap$		$\cup$	

-2 local max

$+2$ local min

(10) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$.