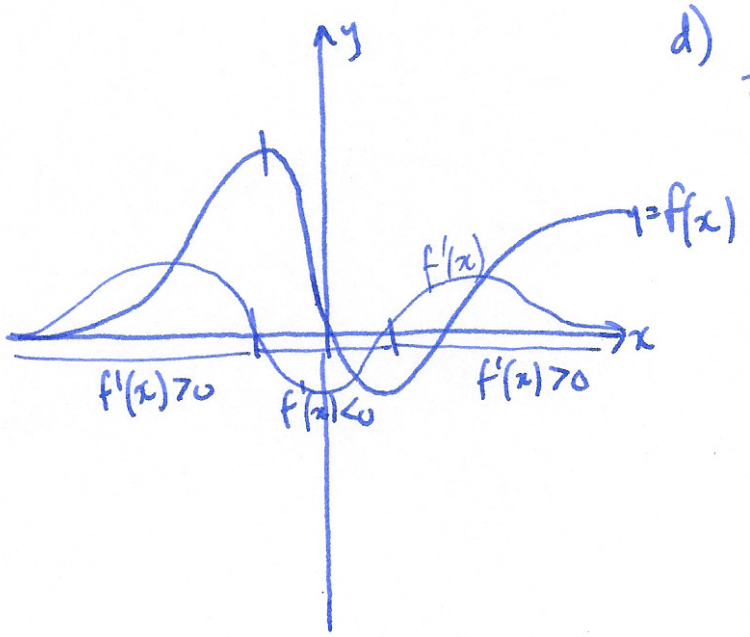
Q1



d) $\lim_{x\to\infty} f(x) = 2$

e) $\lim_{x\to-\infty} f'(x) = 0$.

Q2 / Q3

a) $f(x) = x^3 e^{-4x^2}$ $\quad f'(x) = 3x^2 \cdot e^{-4x^2} + x^3 \cdot -8x\, e^{-8x^2}$

$f''(x) = 6x\, e^{-4x^2} + 3x^2 \cdot e^{-4x^2} \cdot -8x + 3x^2 \cdot -8x\, e^{-8x^2} + x^3 \cdot 64x^2 e^{-8x^2}$

b) $f(x) = \dfrac{\sqrt{2x+1}}{3-\sec(2x)}$ $\quad f'(x) = \dfrac{(3-\sec(2x))\frac{1}{2}(2x+1)^{-1/2}\cdot 2 - \sqrt{2x+1}\cdot(-\sec(2x)\tan(2x)\cdot 2)}{(3-\sec(2x))^2}$

$f''(x) = \dfrac{(3-\sec(2x))^2 \cdot \Big[(-\sec(2x)\tan(2x)\cdot 2)\cdot(2x+1)^{-1/2} + (3-\sec(2x))\cdot(-\frac{1}{2}(2x+1)^{-3/2}\cdot 2) - \circledast \cdot$
$+ \frac{1}{2}(2x+1)^{-1/2}\cdot 2(\sec(2x)\tan(2x)\cdot 2) + \sqrt{2x+1}\cdot(2\sec(2x)\tan(2x)\tan(2x)\cdot 2$
$+ 2\sec(2x)\sec^2(2x)\cdot 2)\Big]$
$\overline{(3-\sec(2x))^4}$

where $\circledast = \big((3-\sec(2x))(2x+1)^{-1/2} + \sqrt{2x+1}\sec(2x)\tan(2x)\cdot 2\big)\cdot 2(3-\sec(2x))\cdot\sec(2x)\tan(2x)\cdot 2$

c) $f(x) = e^{3x\ln(x)}$ $\quad f'(x) = e^{3x\ln(x)}\cdot\left(3\ln(x) + 3x\cdot\frac{1}{x}\right)$

$f''(x) = e^{3x\ln(x)}\left(3\ln(x)+3\right)^2 + e^{3x\ln(x)}\left(\frac{3}{x}\right)$.

d) $f(x) = \ln(\tan(3x))$ $\quad f'(x) = \dfrac{1}{\tan(2x)}\sec^2(3x)\cdot 3 = 3\,\dfrac{\cos(3x)}{\sin(3x)}\dfrac{1}{\cos^2(3x)} = 3\sec(3x)\operatorname{cosec}(3x)$

$f''(x) = 3\sec(3x)\tan(3x)\cdot 3\operatorname{cosec}(3x) + 3\sec(3x)\cdot(-\operatorname{cosec}(3x)\cot(3x)\cdot 3)$.

e) $\tan^{-1}\left(\frac{3}{x^{1/3}}\right) = \tan^{-1}(3x^{-1/3})$ $f'(x) = \frac{1}{1+(3x^{-1/3})^2}\cdot -x^{-4/3} = \frac{-x^{-4/3}}{1+9x^{-2/3}} = (x^{4/3}+9x^{2/3})^{-1}$ (2)

$f''(x) = -(x^{4/3}+9x^{2/3})^{-2}\cdot\left(\frac{4}{3}x^{1/3}+6x^{-1/3}\right)$

f) $f(x) = \cos^{-1}(1-2x)$ $f'(x) = \frac{-1}{\sqrt{1-(1-2x)^2}} = (1-(1-2x)^2)^{-1/2}$

$f''(x) = -\frac{1}{2}(1-(1-2x)^2)^{-3/2}\cdot 2(1-2x)(-2)$

Q4 a) $h(x) = f(x)g(x)$ $h'(x) = f'(x)g(x)+f(x)g'(x)$

$h'(2) = f'(2)g(2)+f(2)g'(2) = 1\cdot 1 + (-2)\cdot(-\frac{1}{2}) = 1+1 = 2$

b) $h(x) = f(g(x))$ $h'(x) = f'(g(x))\cdot g'(x)$

$h'(-1) = f'(g(-1))\cdot g'(-1) = f'(3/2)\cdot g'(-1) = 1\cdot\frac{1}{2} = \frac{1}{2}$

Q5 $4x^2-9y^2=7$ $8x-18yy'=0$ $y' = \frac{8x}{18y} = \frac{4x}{9y} = \frac{8}{9}$

Q6 $x^2y+2x^2y = -\cos(xy)$

$2xy+x^2y'+4xy+2x^2y' = \sin(xy)(y+xy')$

$y'(x^2+3x^2-\sin(xy)\cdot x) = -6xy+\sin(xy)\cdot y$

$y' = \frac{-6xy+\sin(xy)y}{3x^2-\sin(xy)\cdot x}$

Q7 $A = 4\pi r^2$ $\frac{dA}{dt} = 8\pi\frac{dr}{dt}r$ $r=10$

$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ $\frac{dV}{dt}=10$ $10 = 4\pi\cdot 100\frac{dr}{dt}$ $\frac{dr}{dt} = \frac{1}{40\pi}$

$\frac{dA}{dt} = 8\cdot\pi\cdot 10\cdot\frac{1}{40\pi} = 2\ cm^2/sec.$

Q8 $f(x) = x^{1/3}$ $f(64) = 4$

$f'(x) = \frac{1}{3}x^{-2/3}$ $f'(64) = \frac{1}{3\cdot 16} = \frac{1}{48}$

$f(63) \simeq f(64)+f'(64)\cdot(-1) = 4-\frac{1}{48}$

percentage error $= \left|\frac{\sqrt[3]{63} - (4 - 1/48)}{\sqrt[3]{63}}\right| \times 100 = 0.00275\%$

Q9 $f(x) = 2x^3 - 3x^2 - 36x + 4$

$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2)$ critical points $x = -2, 3$.

		-2		3	
$(x-3)$	$-$		$-$		$+$
$(x+2)$	$-$		$+$		$+$
$f'(x)$	$+$	-2	$-$	3	$+$

-2 local max

3 local min.

Q10 $f(x) = x^2 + 2x - 3$

$f'(x) = 2x + 2$ critical point $x = -1$ check $-1, -2, 2$.

$f(-2) = -3$

$f(-1) = -4$ min

$f(2) = 5$ max.