Math 231 Calculus 1 Fall 18 Midterm 1b

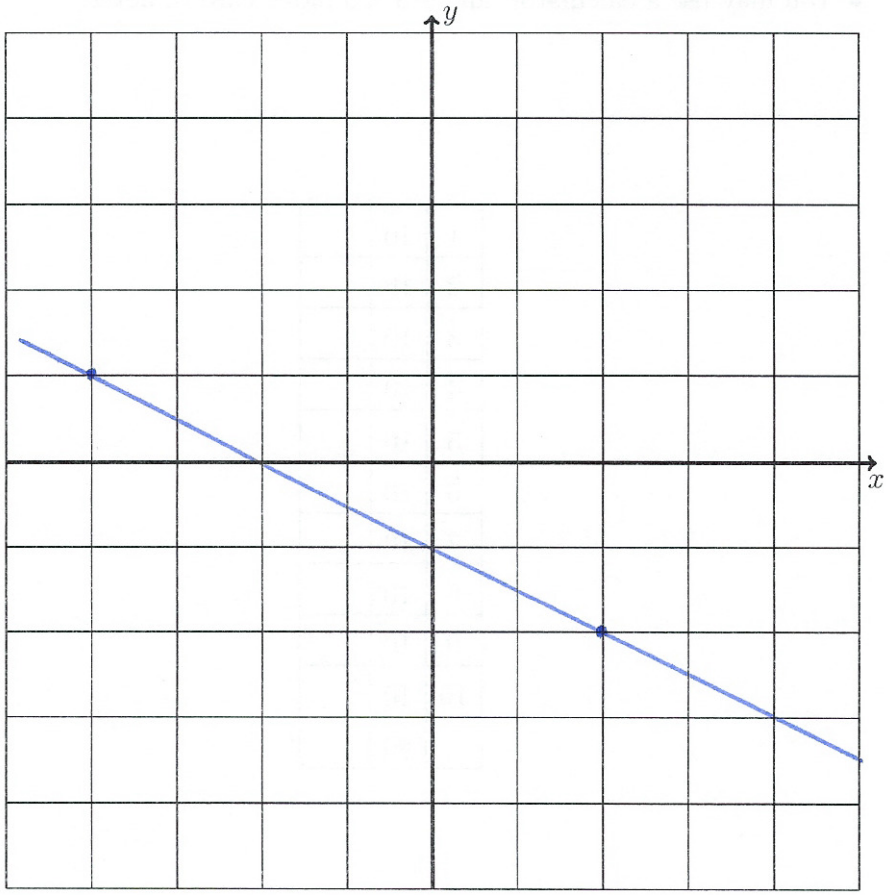
Name: solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(2, -2)$ and $(-4, 1)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



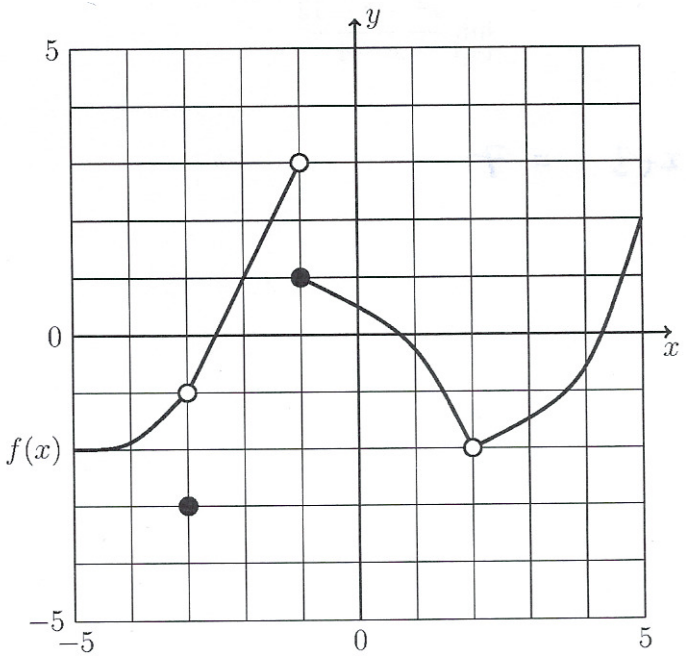
slope $m = \frac{1-(-2)}{-4-2} = \frac{3}{-6} = -\frac{1}{2}$

$y - y_0 = m(x - x_0)$

$y - 1 = -\frac{1}{2}(x - (-4))$

$y = -\frac{1}{2}x - 1$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to -3} f(x)$ -1

(b) $\lim_{x\to 2-} f(x)$ -2

(c) $\lim_{x\to 2+} f(x)$ -2

(d) $\lim_{x\to 2} f(x)$ -2

(e) $\lim_{x\to -1+} f(x)$ 1

(f) $\lim_{x\to -1} f(x)$ DNE

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{x^2 - x - 12}{x - 4}$$

$$\lim_{x\to 4} \frac{(x-4)(x+3)}{x-4} = \lim_{x\to 4} x+3 = 7$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3}$$

$$\lim_{x \to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \lim_{x \to 9} \sqrt{x}+3 = 6$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 + 3x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - x^2 - 3x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h} = \lim_{h \to 0} 2x + h + 3 = 2x + 3$$

(6) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{x+3}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h} = \lim_{h\to 0} \frac{1}{h} \frac{x+3-(x+h+3)}{(x+h+3)(x+3)} = \lim_{h\to 0} \frac{-h}{h(x+h+3)(x+3)}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h+3)(x+3)} = \frac{-1}{(x+3)^2}$$

(7) (10 points) Find the horizontal asymptotes of $f(x) = \dfrac{\sqrt{x^2-4}}{3x-2}$.

$$\lim_{x\to\infty} \frac{\sqrt{x^2-4}}{3x-2} = \lim_{x\to\infty} \frac{\sqrt{1-4/x^2}}{3-2/x} = \frac{1}{3}.$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2-4}}{3x-2} = \lim_{x\to\infty} \frac{\sqrt{x^2-4}}{-3x-2} = \lim_{x\to\infty} \frac{\sqrt{1-4/x^2}}{-3-2/x} = -\frac{1}{3}$$

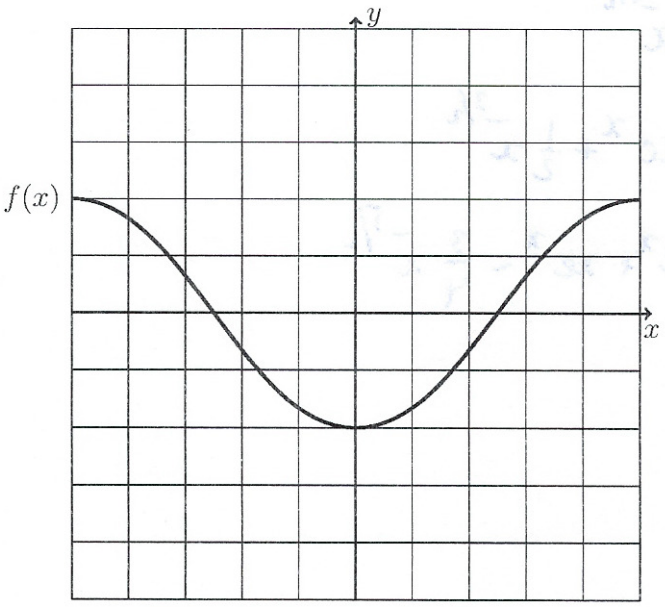
(8) Find the first and second derivatives of $f(x) = xe^x - 1/\sqrt{x}$.

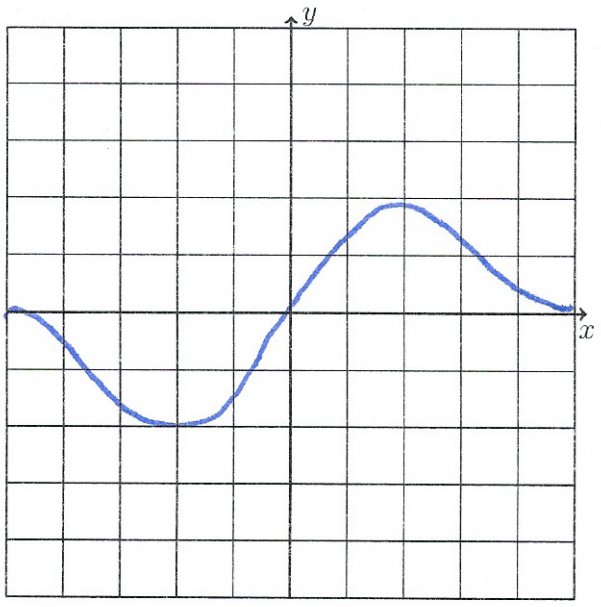
$f(x) = xe^x - x^{-1/2}$

$f'(x) = e^x + xe^x + \frac{1}{2}x^{-3/2}$

$f''(x) = e^x + e^x + xe^x - \frac{3}{4}x^{-5/2}$

(9) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.





(10) (10 points) Sketch the graph of a function for which $f(1) = 2$, f is decreasing for $x < 0$ and increasing for $x > 0$, and $\lim_{x \to \infty} f(x) = 3$.