Math 231 Calculus 1 Fall 18 Midterm 1a

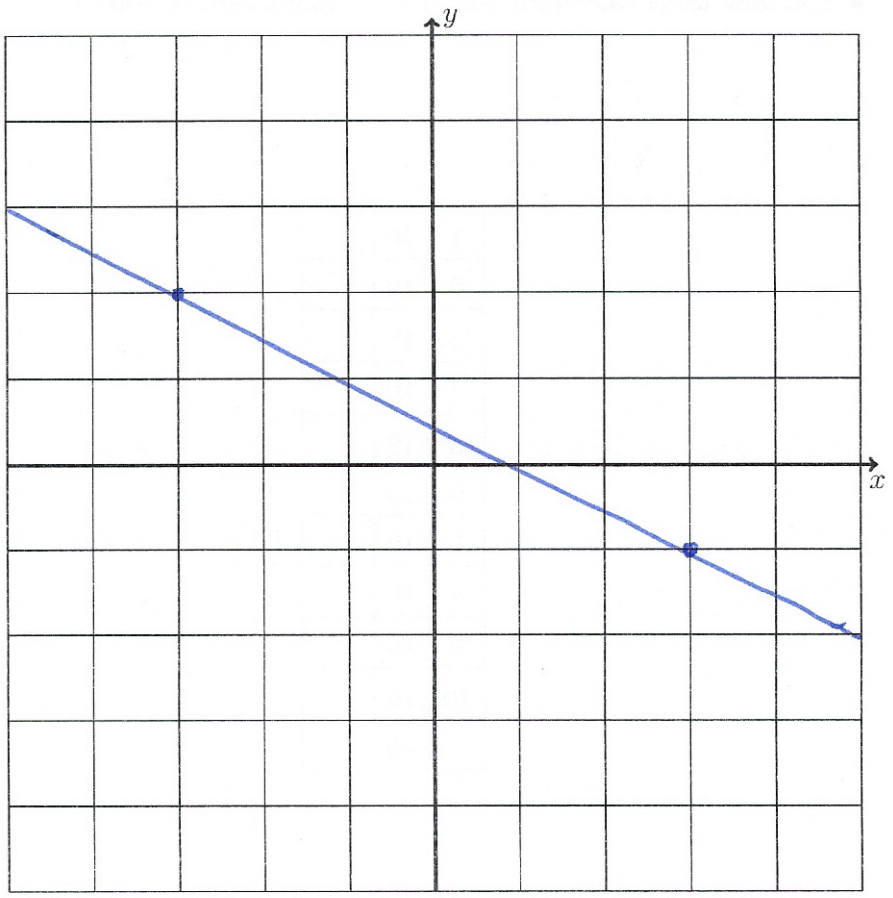
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(3, -1)$ and $(-3, 2)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



slope $m = \dfrac{2-(-1)}{-3-3} = \dfrac{3}{-6} = -\dfrac{1}{2}$

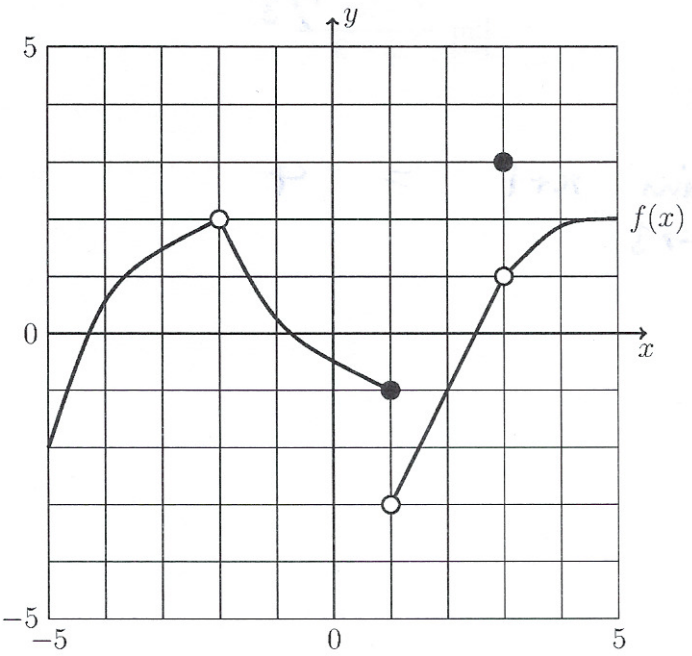
$y - y_0 = m(x - x_0)$

$y - 2 = -\frac{1}{2}(x - (-3))$

$y = -\frac{1}{2}x - \frac{3}{2} + 2$

$y = -\frac{1}{2}x + \frac{1}{2}$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to 3} f(x)$ 1

(b) $\lim_{x\to -2-} f(x)$ 2

(c) $\lim_{x\to -2+} f(x)$ 2

(d) $\lim_{x\to -2} f(x)$ 2

(e) $\lim_{x\to 1+} f(x)$ -3

(f) $\lim_{x\to 1} f(x)$ DNE

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3}$$

$$\lim_{x\to 3} \frac{(x-3)(x+1)}{x-3} = \lim_{x\to 3} x+1 = 4$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$$

$$\lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \lim_{x\to 4} \sqrt{x}+2 = 4$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 2x$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h} = \lim_{h\to 0} 2x+h-2 = 2x-2$$

(6) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{x+2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} = \lim_{h\to 0} \frac{1}{h} \frac{x+2-(x+h+2)}{(x+h+2)(x+2)}$$

$$= \lim_{h\to 0} \frac{-h}{h(x+h+2)(x+2)} = \lim_{h\to 0} \frac{-1}{(x+h+2)(x+2)} = \frac{-1}{(x+2)^2}$$

(7) (10 points) Find the horizontal asymptotes of $f(x) = \dfrac{\sqrt{4x^2-1}}{x+2}$.

$$\lim_{x\to\infty} \frac{\sqrt{4x^2-1}}{x+2} = \lim_{x\to\infty} \frac{\sqrt{4-1/x^2}}{1+2/x} = \frac{2}{1} = 2$$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2-1}}{x+2} = \lim_{x\to\infty} \frac{\sqrt{4x^2-1}}{-x+2} = \lim_{x\to\infty} \frac{\sqrt{4-1/x^2}}{-1+2/x} = -2$$

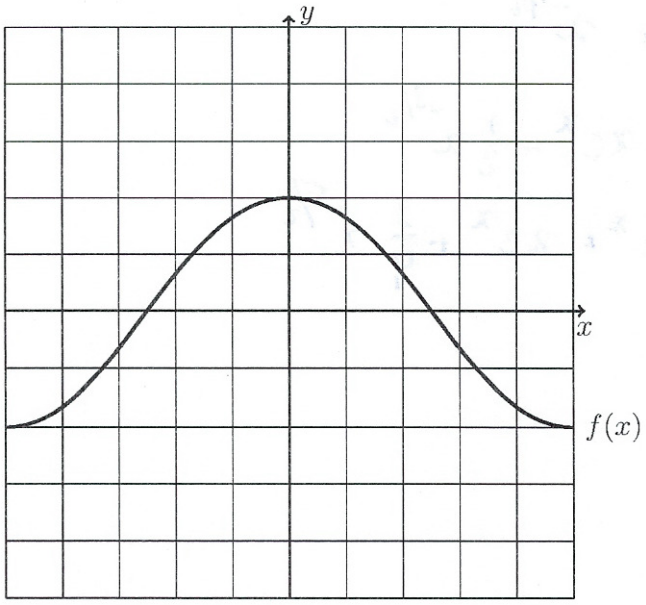
(8) Find the first and second derivatives of $f(x) = xe^x + 1/\sqrt{x}$.

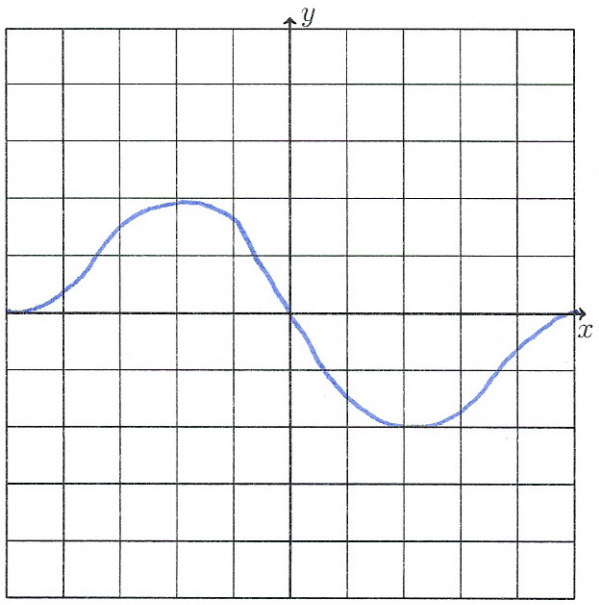
$f(x) = xe^x + x^{-1/2}$

$f'(x) = e^x + xe^x - \frac{1}{2}x^{-3/2}$

$f''(x) = e^x + e^x + xe^x + \frac{3}{4}x^{-5/2}$

(9) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.





(10) (10 points) Sketch the graph of a function for which $f(2) = 1$, f is decreasing for $x > 0$ and increasing for $x < 0$, and $\lim_{x \to \infty} f(x) = -2$.