

Midterm 1 Solutions

①

Q1 a) 3 b) 2 c) DNE d) 2 e) 2 f) -2

Q2 a) $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x + 2} = \lim_{x \rightarrow -2} \frac{(x+2)(x-3)}{x+2} = \lim_{x \rightarrow -2} x - 3 = -5$

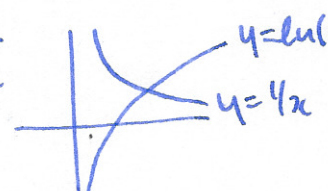
b) $\lim_{x \rightarrow \infty} \frac{\sqrt{4+x^2}}{x-5} = \lim_{x \rightarrow \infty} \frac{\sqrt{1+4/x^2}}{1-5/x} = 1$

c) $\lim_{x \rightarrow 0} \frac{\sin 4x}{5x} = \lim_{x \rightarrow 0} \frac{\sin \theta}{5\theta/4} = \frac{4}{5}$

d) $\lim_{x \rightarrow 2+} \frac{1}{\sqrt{x-2}} - \frac{1}{\sqrt{x^2-4}} = \lim_{x \rightarrow 2+} \frac{\sqrt{x+2}-1}{\sqrt{x^2-4}} = \lim_{x \rightarrow 2+} \frac{1}{\sqrt{x^2-4}} = +\infty$

Q3 $\lim_{x \rightarrow 3-} x - \frac{2}{x-2} = 3 - 2 = 1$ $\lim_{x \rightarrow 3+} \frac{-\cos(\pi x)}{x} = \frac{1}{3}$ $1 \neq \frac{1}{3}$ no value of c works.

Q4 average s.o.c. $\frac{s(4) - s(3)}{1} = 4\pi \cdot 16 - 4\pi \cdot 9 = 28\pi$

Q5  $y = \ln(x)$ $y = 1/x$ $f(x) = 1/x - \ln(x)$ $f(1) = 1/1 - \ln(1) = 1 > 0$ $f(e) = 1/e - 1 < 0$ $\Rightarrow f(x) = 0$ in $[1, e]$.

Q6 a) $(3x - 4x^{1/2} - 7x^{1/2} + x^{-1/3})' = 3 - 4 \cdot \frac{1}{2} x^{-1/2} - 7 \cdot \frac{1}{2} x^{-1/2} - \frac{1}{3} x^{-4/3}$

b) $(-3x^2 e^x)' = -6x e^x - 3x^2 e^x$

c) $(\frac{x-4}{x+4})' = \frac{(x+4) \cdot 1 - (x-4) \cdot 1}{(x+4)^2} = \frac{8}{(x+4)^2}$

Q7 a) $-528\pi^{10} + \frac{7}{4} x^{-3/2} + \frac{4}{9} x^{-7/3}$

b) $-6e^x - 6xe^x - 6\pi e^x - 3\pi^2 e^x$

c) $\frac{(x+4) \cdot 0 - 8 \cdot 2(x+4)}{(x+4)^4} = -\frac{16}{(x+4)^3}$

Q8 a) $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \frac{x+1 - (x+h+1)}{(x+h+1)(x+1)} = \lim_{h \rightarrow 0} \frac{1}{h} \frac{-h}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2}$

b) $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \frac{\sqrt{x+2} - \sqrt{x+h+2}}{\sqrt{(x+h+2)(x+2)}} \cdot \frac{\sqrt{x+2} + \sqrt{x+h+2}}{\sqrt{x+2} + \sqrt{x+h+2}}$
 $= \lim_{h \rightarrow 0} \frac{1}{h} \frac{x+2 - x-h-2}{\sqrt{(x+h+2)(x+2)}(\sqrt{x+2} + \sqrt{x+h+2})} = \lim_{h \rightarrow 0} \frac{-1}{\sqrt{(x+h+2)(x+2)}(\sqrt{x+2} + \sqrt{x+h+2})} = \frac{-1}{(x+2)\sqrt{x+2}} = \frac{-1}{2(x+2)^{3/2}}$