Math 231 Calculus 1 Fall 18 Sample Final

(1) Differentiate the following functions. Do not simplify your answers.

   (a) $-2x^7 + 2x^{2/3} - \tan(4x)$

   (b) $f(x) = \dfrac{\ln(x^2 - x)}{2x + 1}$

   (c) $f(x) = e^{-2x} \tan(3x + 1)$

   (d) $f(x) = \sqrt[4]{e^{-\cos(4x)} + 1}$

(2) Evaluate the following integrals.

   (a) $\displaystyle \int x^2 + \sin(x) - e^x \; dx$

   (b) $\displaystyle \int \frac{(x-2)^2}{\sqrt[3]{x^2}} \; dx$

   (c) $\displaystyle \int_0^{\pi/4} \cos^2(3x) \sin(3x) \; dx$

   (d) $\displaystyle \int \frac{1}{4 + x^2} \; dx$

(3) Note: the possible answers for limits are a number, $+\infty$, $-\infty$ or "does not exist" (DNE). Justify your answers.

   (a) Find $\displaystyle \lim_{x \to 4} \frac{x^2 - 5x + 4}{x - 4}$.

   (b) Find $\displaystyle \lim_{x \to 0} \frac{\sin 3x}{e^{2x} - 1}$.

   (c) Find $\displaystyle \lim_{x \to 0+} x^{\cos(x) - 1}$.

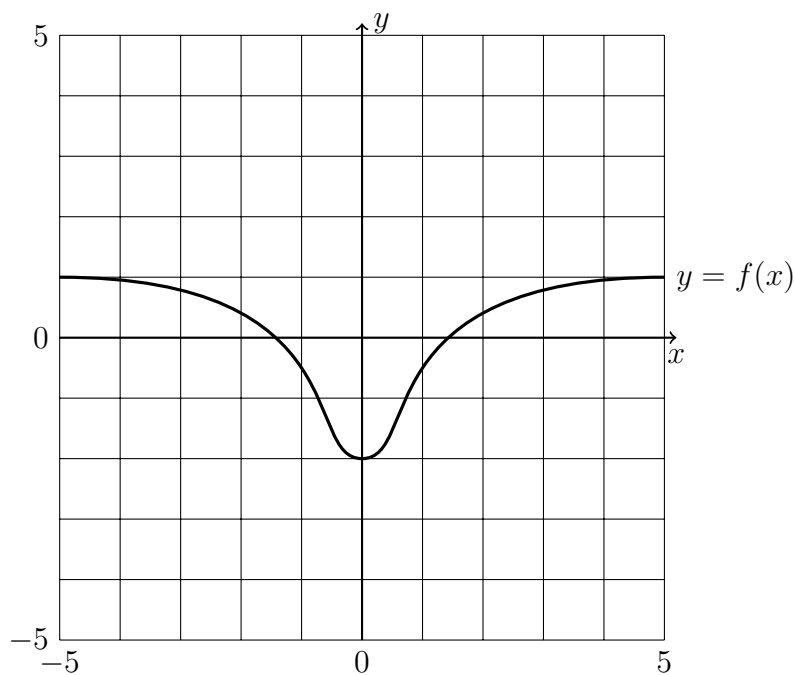   (d) Find $\displaystyle \lim_{x \to 0} \frac{1}{x} - \frac{1}{e^x - 1}$.

(4) Consider $f(x) = x^3 + 6x$.

   (a) Find the derivative of $f(x)$, and find the critical points for $f(x)$.

   (b) Give the interval(s) for which f is increasing.

(c) Give the intervals for which f is concave up, and for which it is concave down.

(d) Decide which critical points are maxima, minima, or neither.

(e) Sketch the graph of $f(x)$.

(5) Consider the function $f(x)$ defined by the following graph.

(a) Label all regions where $f(x) < 0$.

(b) Label all regions where $f'(x) > 0$.

(c) Sketch a graph of $f'(x)$ on the figure.

(6) Consider $f(x) = \dfrac{2}{x-4}$.

(a) Sketch the graph of $f(x)$ showing any asymptotes.

(b) Find the slope of the tangent line at $x = 2$, and write down the equation for the tangent line.

(c) Sketch the tangent line at $x = 2$ on your graph.

(7) Let $f(x) = x^2 + 2x$. Find the derivative *using the limit definition of the derivate.* Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation $x^3y + 3x^2y^2 - xy^2 = 6$ at the point $(-2, 1)$.

(9) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.
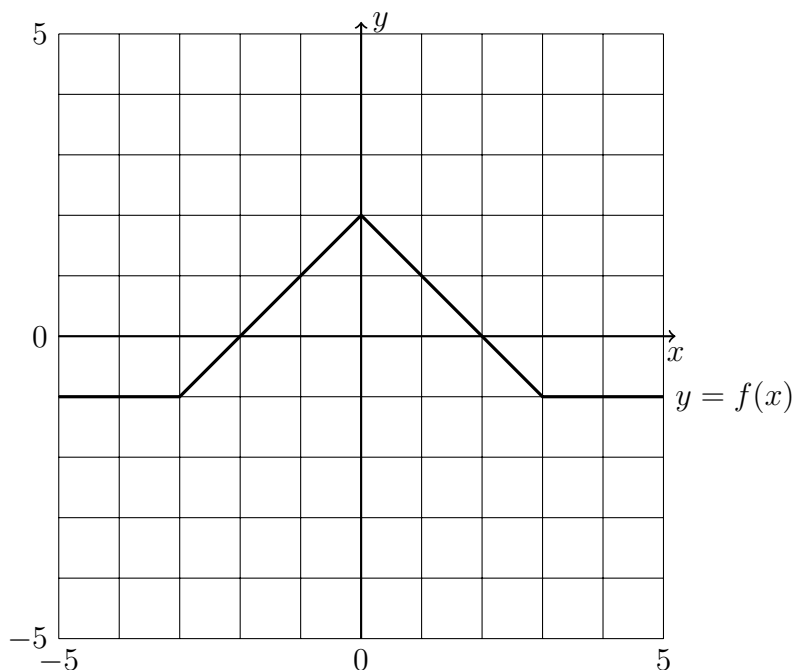
(10) A region in the plane is bounded by the x-axis, the graph $y = 16 - x^2$, and the lines $x = -1$ and $x = 1$.
   (a) Sketch the region (shading it in) and label the boundaries.
   (b) Find the area of the region.

(11) You blow up a spherical balloon at the rate of $4\text{in}^3/\text{s}$. How fast is the surface area growing when $r = 6\text{in}$? (The volume of a sphere is $V = \frac{4}{3}\pi r^3$, and the surface area is $A = 4\pi r^2$.)

(12) Use linear approximation to estimate $\sqrt[3]{26}$. Use you calculator to find the exact value, and find the absolute and percentage errors.

(13) You wish to build a running track in the shape of a rectangle with two semi-circular ends. If the running rack should have length 800m, what shape minimizes the area?