Solutions

Q1 a) $-14x^6 + \frac{4}{3}x^{-1/3} - \sec^2(4x).4$

b) $\dfrac{(2x+1).\frac{1}{x^2-x}.(2x-1) - 2\ln(x^2-x)}{(2x+1)^2}.$

c) $-2e^{-2x}\tan(3x+1) + e^{-2x}.\sec^2(3x+1).3.$

d) $\frac{1}{4}\left(e^{-\cos(4x)}+1\right)^{-3/4}.\ e^{-\cos(4x)}.\ -\sin(4x).4.$

Q2 a) $\frac{1}{3}x^3 - \cos(x) - e^x + c$

b) $\displaystyle\int \frac{x^2-4x+4}{x^{2/3}}\,dx = \int x^{4/3} - 4x^{1/3} + 4x^{-2/3}\,dx = \frac{3}{7}x^{7/3} - 3x^{4/3} + 12x^{1/3} + c$

c) $\displaystyle\int_0^{\pi/4} \cos^2(3x)\sin(3x)\,dx$ $\quad u = \cos(3x)$, $\frac{du}{dx} = -\sin(3x).3$ $\quad \displaystyle\int_1^{\cos(3\pi/4) = -1/\sqrt{2}} u^2 \sin(3x)\,\frac{dx}{du}\,du$

$\displaystyle = \int_1^{-1/\sqrt{2}} u^2.\sin(3x).\frac{1}{-\sin(3x).3}\,du = \int_1^{-1/\sqrt{2}} -\frac{1}{3}u^2\,du = \left[-\frac{1}{9}u^3\right]_1^{-1/\sqrt{2}} = -\frac{1}{9}.\frac{-1}{2\sqrt{2}} + \frac{1}{9}$

$= \frac{1}{9}\left(1 - \frac{1}{2\sqrt{2}}\right).$

d) $\displaystyle\int \frac{1}{4+x^2}\,dx = \frac{1}{4}\int \frac{1}{1+(x/2)^2}\,dx$ $\quad u = x/2$, $\frac{du}{dx} = \frac{1}{2}$ $\quad \displaystyle\frac{1}{4}\int \frac{1}{1+u^2}\frac{dx}{du}\,du$

$\displaystyle = \frac{1}{4}\int \frac{1}{1+u^2}\,2\,du = \frac{1}{2}\tan^{-1}(u) + c = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c.$

Q3

a) $\displaystyle\lim_{x\to 4} \frac{x^2-5x+4}{x-4} \overset{L'H}{=} \lim_{x\to 4} \frac{2x-5}{1} = 3.$

b) $\displaystyle\lim_{x\to 0} \frac{\sin 3x}{e^{2x}-1} \overset{L'H}{=} \lim_{x\to 0} \frac{3\cos 3x}{2e^{2x}} = \frac{3}{2}$

c) $\displaystyle\lim_{x\to 0+} x^{\cos(x)-1} = \lim_{x\to 0+} e^{\ln(x)(\cos x - 1)} = e^{\lim_{x\to 0+}\ln(x)(\cos x - 1)}$

$\lim_{x\to 0^+} \frac{\ln(x)}{1/(\cos x - 1)} \overset{l'H}{=} \lim_{x\to 0^+} \frac{1/x}{-(\cos x - 1)^{-2}\cdot -\sin x} = \lim_{x\to 0^+} \frac{(\cos x - 1)^2}{x \sin x}$

$\overset{l'H}{=} \lim_{x\to 0^+} \frac{2(\cos x - 1)\cdot(-\sin x)}{\sin x + x\cos x} = \lim_{x\to 0} \frac{-2\sin x\cos x + 2\sin x}{\sin x + x\cos x} \overset{l'H}{=} \lim_{x\to 0^+} \frac{-2\cos^2 x + 2\sin^2 x + 2\cos x}{\cos x + \cos x - x\sin x}$

$= 0$ so $\lim_{x\to 0} x^{\cos(x)-1} = x^0 = 1$

~~Q4 f(x) = x~~ d) $\lim_{x\to 0} \frac{1}{x} - \frac{1}{e^x - 1} = \lim_{x\to 0} \frac{e^x - 1 - x}{x(e^x - 1)} \overset{l'H}{=} \lim_{x\to 0} \frac{e^x - 1}{e^x - 1 + xe^x}$

$\overset{l'H}{=} \lim_{x\to 0} \frac{e^x}{e^x + e^x + xe^x} = \frac{1}{2}$

Q4 $f(x) = x^3 + 6x$

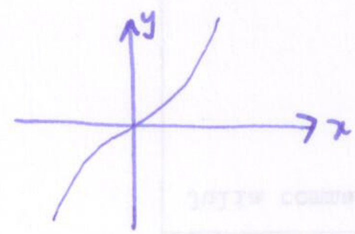
a) $f'(x) = 3x^2 + 6$ critical pts solve $f'(x) = 0$: $\cancel{x^2 \neq}$ $x^2 + 2 > 0$ no critical pnts

b) $(-\infty, \infty)$

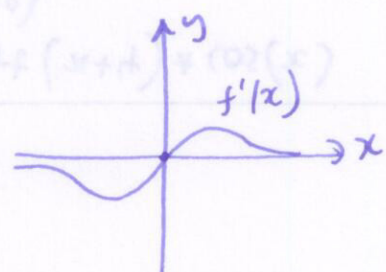
c) $f''(x) = 6x$ concave up $(0, \infty)$ concave down $(-\infty, 0)$

d) N/A

e)

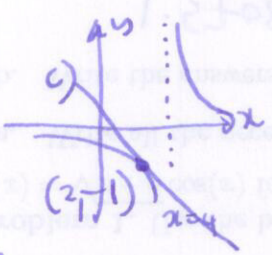


Q5 a) $(-3/2, 3/2)$ b) $(0, 5)$ c)



Q6 $f(x) = \frac{2}{x-4}$

a)



b) $f'(x) = -2(x-4)^2$ $f'(2) = -2(-2)^2 = -8$ $y + 1 = -8(x - 2)$

Q7 $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $f(x) = x^2 + 2x$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 + 2(x+h) - x^2 - 2x}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$= \lim_{h\to 0} 2x + h + 2 = 2x+2.$$

Q8 $x^3 y + 3x^2y^2 - xy^2 = 6$

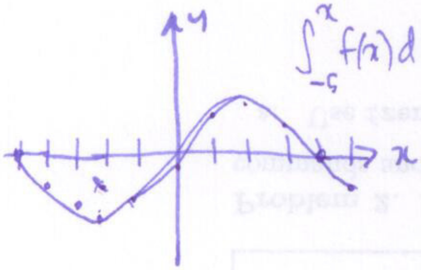
$$3x^2y + x^3y' + 6xy^2 + 3x^2 2yy' - 2xy - x2yy' = 0$$

$$3x^2y + 6xy^2 - 2xy + y'(x^3 + 6x^2y - 2xy) = 0$$

at $(-2,1)$ $\quad 12 + -12 + 2 + y'(-8 + 24 + 4) = 0 \qquad y' = \frac{-2}{20} = -\frac{1}{10}.$

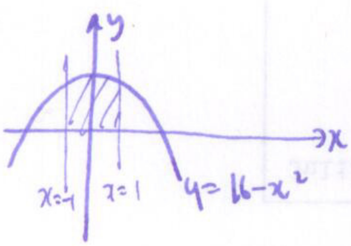
$$y - 1 = -\frac{1}{10}(x+2)$$

Q9



$$\int_{-5}^{x} f(x)\,dx.$$

Q10



$$\int_{-1}^{1} 16 - x^2\,dx = \left[16x - \frac{1}{3}x^3\right]_{-1}^{1} = 16 - \frac{1}{3} - \left(-16 + \frac{1}{3}\right)$$

$$= 32 - \frac{2}{3} = \frac{94}{3}.$$

Q11

$V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4 \qquad r=6: \ \frac{dr}{dt} = \frac{1}{36\pi}\ \text{in/s}.$

$A = 4\pi r^2 \qquad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$$\frac{dA}{dt} = 8\pi \cdot 6 \frac{1}{36\pi} = \frac{8}{6} = \frac{4}{3}\ \text{in}^2/\text{s}$$

Q12 $f(x) = x^{1/3} \qquad f(27) = 3.$

$f'(x) = \frac{1}{3}x^{-2/3}$

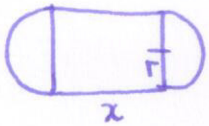
$$f'(26) \approx f(27) + f'(27)\cdot(-1)$$

$$= 3 + \frac{1}{3\cdot 9}\cdot -1 = 3 - \frac{1}{27} \approx 2.962963$$

abs error $= |\sqrt[3]{26} - 2.962963| = 0.0004668943$

percentage error $= \frac{\text{abs error}}{\text{correct}} \times 100 \approx 0.01576017\%$

Q13



$2x + 2\pi r = 800 \qquad x = 400 - \pi r$

$A = 2xr + \pi r^2$

$A = 2(400 - \pi r)r + \pi r^2 = 800r - \pi r^2$

$\frac{dA}{dr} = 800 - 2\pi r = 0 \qquad r = \frac{400}{\pi} \qquad x = 400 - 400 = 0$.