

Solutions

(1)

Q1 a) $-14x^6 + \frac{4}{3}x^{-1/3} - \sec^2(4x) \cdot 4$

b) $\frac{(2x+1) \cdot \frac{1}{x^2-x} \cdot (x-1) - 2\ln(x^2-x)}{(2x+1)^2}$

c) $-2e^{-2x} \tan(3x+1) + e^{-2x} \cdot \sec^2(3x+1) \cdot 3$

d) $\frac{1}{4} (e^{-\cos(4x)} + 1)^{-3/4} \cdot e^{-\cos(4x)} \cdot -\sin(4x) \cdot 4$

Q2 a) $\frac{1}{3}x^3 - \cos(x) - e^x + C$

b) $\int \frac{x^2 - 4x + 4}{x^{2/3}} dx = \int x^{4/3} - 4x^{1/3} + 4x^{-2/3} dx = \frac{3}{7}x^{7/3} - 3x^{4/3} + 12x^{1/3} + C$

c) $\int_0^{3\pi/4} \cos^2(3x) \sin(3x) dx$
 $u = \cos(3x)$
 $\frac{du}{dx} = -\sin(3x) \cdot 3$
 $\cos(3\pi/4) = -1/\sqrt{2}$
 $\int_1^{-1/\sqrt{2}} u^2 \sin(3x) \frac{dx}{du} du$
 $= \int_1^{-1/\sqrt{2}} u^2 \sin(3x) \cdot \frac{1}{-\sin(3x) \cdot 3} du = \int_1^{-1/\sqrt{2}} -\frac{1}{3} u^2 du = \left[-\frac{1}{9} u^3 \right]_1^{-1/\sqrt{2}} = -\frac{1}{9} \cdot \frac{-1}{2\sqrt{2}} + \frac{1}{9}$
 $= \frac{1}{9} (1 - \frac{1}{2\sqrt{2}})$

d) $\int \frac{1}{4+x^2} dx = \frac{1}{4} \int \frac{1}{1+(x/2)^2} dx$
 $u = x/2$
 $\frac{du}{dx} = \frac{1}{2}$
 $\frac{1}{4} \int \frac{1}{1+u^2} \frac{dx}{du} du$
 $= \frac{1}{4} \int \frac{1}{1+u^2} 2 du = \frac{1}{2} \tan^{-1}(u) + C = \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$

Q3 a) $\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x - 4} \stackrel{1/0}{=} \lim_{x \rightarrow 4} \frac{2x - 5}{1} = 3$

b) $\lim_{x \rightarrow 0} \frac{\sin 3x}{e^{2x} - 1} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{3\cos 3x}{2e^{2x}} = \frac{3}{2}$

c) $\lim_{x \rightarrow 0+} x^{\cos(x)-1} = \lim_{x \rightarrow 0+} e^{\ln(x)(\cos(x)-1)} = \lim_{x \rightarrow 0+} \ln(x)(\cos(x)-1)$

$$\lim_{x \rightarrow 0^+} \frac{\ln(x)}{1/\cos x - 1} \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{1/x}{-(\cos x - 1)^{-2} \cdot -\sin x} = \lim_{x \rightarrow 0^+} \frac{(\cos x - 1)^2}{x \sin x}$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{2(\cos x - 1) \cdot (-\sin x)}{\sin x + x \cos x} = \lim_{x \rightarrow 0} \frac{-2 \sin x \cos x + 2 \sin x}{\sin x + x \cos x} \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{-2 \cos^2 x + 2 \sin^2 x + 2 \cos x}{\cos x + \cos x - x \sin x}$$

$$= 0 \quad \text{so } \lim_{x \rightarrow 0} x^{(\cos x) - 1} = x^0 = 1$$

Q4 ~~f(x) = x~~ d) $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{e^x - 1} = \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x(e^x - 1)} \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{e^x - 1 + x e^x}$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x}{e^x + e^x + x e^x} = \frac{1}{2}$$

Q4 $f(x) = x^3 + 6x$

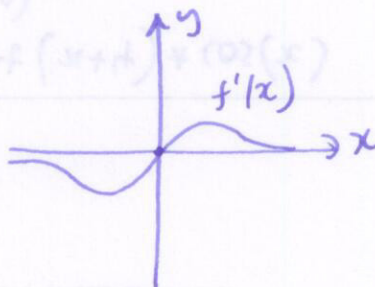
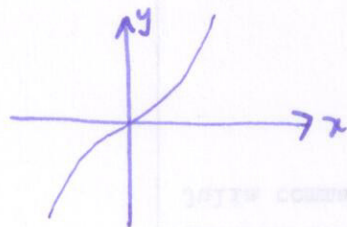
a) $f'(x) = 3x^2 + 6$ critical pt solve $f'(x) = 0$: ~~$x^2 + 2 \geq 0$~~ no critical points

b) $(-\infty, \infty)$

c) $f''(x) = 6x$ concave up $(0, \infty)$ concave down $(-\infty, 0)$

d) N/A

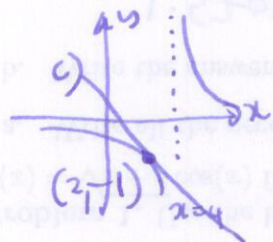
e)



Q5 a) $(-3/2, 3/2)$ b) $(0, 5)$ c)

Q6 $f(x) = \frac{2}{x-4}$

a)



b) $f'(x) = -2(x-4)^{-2}$ $f'(2) = -2(-2)^{-2} = -8$ $y+1 = -8(x-2)$

Q7 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f(x) = x^2 + 2x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) - x^2 - 2x}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h + 2 = 2x + 2.$$

Q8 $x^3y + 3x^2y^2 - xy^3 = 6$

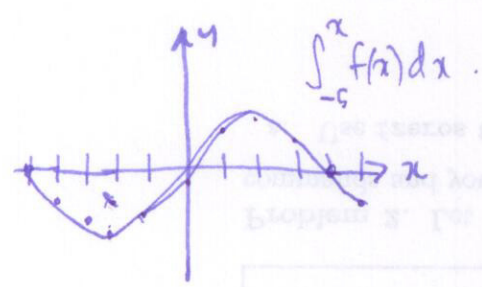
$$3x^2y + x^3y' + 6xy^2 + 3x^2y^2y' - 2xy - x^2yy' = 0$$

$$3x^2y + 6xy^2 - 2xy + y'(x^3 + 6x^2y - 2xy) = 0$$

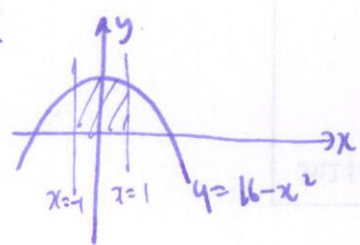
at $(-2, 1)$ $12 + -12 + 2 + y'(-8 + 24 + 4) = 0$ $y' = \frac{-2}{20} = -\frac{1}{10}$

$$y - 1 = -\frac{1}{10}(x + 2)$$

Q9



Q10



$$\int_{-1}^1 (16 - x^2) dx = \left[16x - \frac{1}{2}x^3 \right]_{-1}^1 = 16 - \frac{1}{2} - \left(-16 + \frac{1}{2} \right)$$

$$= 32 - 1 = \frac{94}{2} = 47$$

Q11



$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4$$

$$r = 6 : \frac{dr}{dt} = \frac{1}{36\pi} \text{ in/s}$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi \cdot 6 \cdot \frac{1}{36\pi} = \frac{8}{6} = \frac{4}{3} \text{ in}^2/\text{s}$$

Q12

$$f(x) = x^{1/3}$$

$$f'(x) = \frac{1}{3}x^{-2/3}$$

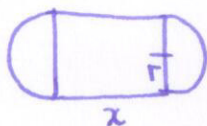
$$f(27) = 3$$

$$f'(26) \approx f'(27) + f''(27) \cdot (-1)$$

$$\text{abs error} = | \sqrt[3]{26} - 2.962963 | = 0.0004669943 + \frac{1}{2 \cdot 9} \cdot -1 = 3 - \frac{1}{27} \approx 2.962963$$

$$\text{percent error} = \frac{\text{abs error}}{\text{correct}} \times 100 \approx 0.01574017\%$$

Q13



$$2x + 2\pi r = 800$$

$$x = 400 - \pi r$$

(4)

$$A = 2\pi r + \pi r^2$$

$$A = 2(400 - \pi r)r + \pi r^2 = 800r - \pi r^2$$

$$\frac{dA}{dr} = 800 - 2\pi r = 0 \quad r = \frac{400}{\pi} \quad x = 400 - 400 = 0$$

Write comments:

1. Use zeros to find all inflection points of $f(x)$; i.e. points where $f''(x) = 0$

2. Use zeros to find all inflection points of $f(x)$; i.e. points where $f''(x) = 0$

conclude and find intervals where f is concave up/down

Problem 3. Let $f(x) = \cos(x) \sin(x+1)$ for $0 \leq x \leq 10$. Write down the value

Write comments:

1. Write the intervals where f is concave up/down

2. Write on the interval $[0, 10]$ where f is concave up/down

$f(x) = \sqrt{x+2} \cos(x)$ for $0 \leq x \leq 10$

Problem 4. Use the first-derivative test to find all zeros of

NAME:

Don't use any other value of x except $x=0$ - no other values

Write your answer