Math 229 Calculus Computer Lab Fall 17 Final b

Name: Solutions

- I will count your best 6 of the following 8 questions.
- You may only use `julia` during this exam. No calculators or cell phones.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
	60	

Final	
Overall	

(1) Convert the following `julia` expressions to standard mathematical expressions. Use parentheses if necessary to clearly indicate the order of operations:

(a) `x-y/x+y/x`

$$x - \frac{y}{x} + \frac{y}{x}$$

(b) `sin(1/3*x^2)/3x^2`

$$\frac{\sin\left(\frac{x^2}{3}\right)}{3x^2}$$

(2) Find all solutions (to at least 4 decimal places) to the equation

$$\frac{x}{x+1}\cos^2(x+1) = \frac{x}{3} - 18.$$

Write down the `julia` command you use.

f(x) = x/(x+1)*cos(x+1)^2 - x/3 + 18

plot(f, -10, 100)

fzeros(f, 40, 70)

answers: 54.0017
54.3061
56.1106

(3) Use `julia` to find

$$\lim_{x\to 0} \frac{\tan(3x^2) - 3\sin^2(x)}{e^{-2x^4} - 1},$$

by any method; write both the `julia` commands and your answers.

```
f(x) = ( tan(3*x^2) - 3*sin(x)^2 ) / ( exp(-2*x^4) - 1 )

limit(f, 0)
```

answer : $-\frac{1}{2}$

(4) Consider the function $f(x) = (x^2 - x)e^{-x^2}$. Use `julia` to find all the critical points; write both the `julia` commands and your answers.

```
f(x) = (x^2-x) * exp(-x^2)

fzeros(f', -5, 5)
```

answers:

-0.854638

0.403032

1.45161

$0 \leq x \leq 8$

(5) Consider a function $f(x)$ for which $f'(x) = 4\sin(x) + x + 1$. Use `julia` to find all the intervals on which the function is concave up; write both the `julia` commands and your answers.

```
f(x) = 4sin(x)+x+1
plot(f,0,8)
fzeros(f',0,8):    1.82348
                   4.45971
```

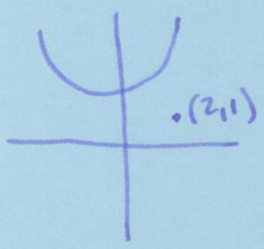
concave up $\leftrightarrow$ 2nd derivative +ve
first derivative increasing

$(0, 1.82348) \cup (4.45971, 8)$

(6) Use the built in Newton's method `newton` to find all zeros of $f(x) = \log(x) + 2\cos(x)$; write both the `julia` commands and your answers.

```
f(x) = log(x)+2*cos(x)
plot(f,0,20)
newton(f,0.2)      0.13793050341719645
newton(f,2)        1.8965431395663408
newton(f,4)        3.9524562369160943
```

(7) Find the closest point on the parabola $y = x^2 + 1$ to the point $(2, 1)$. How far away is it? Write both the `julia` commands and your answers.



$$d = \sqrt{(x-2)^2 + (x^2+1-1)^2}$$

```
d(x) = sqrt( (x-2)^2 + (x^2)^2 )

plot(d,-10,10)

fzeros(d',0,2) :   x = 0.835122

d(fzeros(d',0,2))   distance = 1.3576993861022464
```

(8) Use `julia` to find the area under the curve of

$$f(x) = \frac{e^{-x}}{x+1}$$

between 2 and 4. Write both the `julia` commands and your answers.

```
f(x) = exp(-x)/(x+1)

quadgk(f, 2, 4)

area =  0.032347861797010 1
```